5-2009

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ADULT ESTIMATION, EYE MOVEMENTS AND MATH ANXIETY

by

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Bachelor of Arts
University of Nevada, Las Vegas
2006

A thesis submitted in partial fulfillment
of the requirements for the

Master of Arts Degree in Psychology
Department of Psychology
College of Liberal Arts

Graduate College
University of Nevada, Las Vegas
May 2009

UNLV
UNIVERSITY OF NEVADA LAS VEGAS

Thesis Approval
The Graduate College
University of Nevada, Las Vegas

April 13, 2009

The Thesis prepared by

Robert Durette

Entitled

Adults' Estimation, Eye Movements and Math Anxiety

is approved in partial fulfillment of the requirements for the degree of

Master of Arts in Psychology

[redacted]
Examination Committee Chair

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Dean of the Graduate College

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Examination Committee Member

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Examination Committee Member

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1017-53

ABSTRACT

Adult Estimation, Eye Movements and Math Anxiety

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In this experiment the estimation ability of college undergraduates was examined using a number line task, with lines numbered 0-to-100, 0-to1,000 and 0-to-723 presented on a computer monitor. Previous research on kindergarteners' through 6th graders' ability to estimate showed a progression from a logarithmic mental representation of numbers to a linear mental number line. Children's ability to estimate was found to correlate strongly with math achievement. We used this task to examine the hypothesis that remnants of the underlying logarithmic number line representation persist into adulthood despite formal educational experience with the number system (e.g. Dehaene, 1997). 0 to 723 number lines were presented to add novel mid- and end-points, due to previous research showing that the mid-points of 0 to 100 and 0 to 1,000 lines were used as reference points. Accuracy of estimates and reaction times were analyzed for each subject along with eye-movements in order to help define strategy use in performing the task. In particular, we captured participants' eye fixations within several defined regions of interest along the number lines as a way of identifying their estimation strategies. We presented overlays of fixations and saccades to show exemplars of the several strategies

we have observed. Statistical analysis of these strategies was discussed and a comparison of strategy use with estimation accuracy was examined.

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ACKNOWLEDGEMENTS

First, I would like to take this opportunity to thank my advisor, Mark, for all of your patience and encouragement. Your guidance in this project is a major reason for its' successful completion. I am looking forward to working with you toward earning my Ph.D.

Secondly, I want to thank my parents Thomas and Linda Durette. I have always known that I could call on you, whatever the obstacle. Your belief in me and your constant support has lead to this.

My wife Lisa, who has shown amazing patience and advice, I thank you and will always love you.

With every research project, this was a group effort. I would like to thank my lab mates: Jeremy Krause, Michelle Guillaume, Alex Moore and Nathan Rudig.

CHAPTER 1

INTRODUCTION

People make quantitative estimates on a daily basis. Adults use estimation in a myriad of tasks: currency (how much does this car cost?), temporally (how much longer until lunch time?), distance (how far is my flight to the east coast?), and combinations like cost and distance (was this airfare to the east coast a bargain?) are just a few examples of estimation activities. Adults may take their estimation ability for granted, but with most abilities there is growth and improvement over time. In this domain, adults tend to use a more mature and accurate method of estimating when compared to children. Children estimate, but much less accurately, and they use multiple, sometimes inefficient mental representations as found by Siegler and associates (Booth & Siegler, 2006; Siegler & Booth, 2004; Siegler & Opfer, 2003).

Unlike most other areas of mathematical development, estimation has just recently begun to be studied, which may be due to the nature of estimation tasks. We began above with a number of estimating situations, each requiring a different process or strategy, thereby creating a difficult problem for researchers. The main characteristic of an estimation problem that makes it difficult to study is that various contextual variables are involved. The estimation tasks above involve multiple contexts, such as: what kind of car? What is the duration of an hour? How far is a mile? Results may be influenced positively or negatively by the subject's knowledge of the contextual variables, not by the subject's actual estimation ability.

To avoid this difficulty, Siegler and Opfer (2003) selected an estimation task that removed almost all contexts, and was readily used in classrooms and daily life, a simple number line based task. A number line task seeks to examine an individual's estimation ability in a pure, numerical context. Siegler and associates have used a line with 0 on one end and higher numerical value on the other end (either 10, 100 or 1,000) in order to establish developmental trends and mental representations of individuals from kindergarten up to adulthood. Seigler has repeatedly shown a shift of mental representations on a 0 to 100 number line by first graders using a logarithmic (an inaccurate) mental representation to second graders using a linear (more accurate) mental representation when estimating on a number line.

A linear representation of a number line has a central characteristic: equal intervals. In a linear number line, each number is an equal distance from the next number. Specifically, the distance between digits 1 and 2 is the same as the distance between 2 and 3 and so on. In contrast to a logarithmic 0 to 100 number line, the distance between 2 numbers below the midpoint of 50 of a linear number line is the same distance between 2 similar numbers above the midpoint.

In contrast, a logarithmic representation of a number line has two opposing qualities: compression and expansion. The lower digits of a set of numbers that is viewed as logarithmic would be expanded or overestimated on a number line, while the higher digits would be compressed. The distance between two digits in the lower half of a given number set would be greater than two commensurate digits in the upper half of the same number set, which means the greatest distance between two digits would be between the lowest two digits. The distance between numbers in a number set decreases as the digits themselves increase, or conversely, as digits increase in magnitude on the number line, the distance between the numbers decreases. For example, the distance between digits 1 and 2 would be greater than 2 and 3, the distance between 2 and 3 would be greater than 3 and 4, and so on. As the length that the lower digits occupy on the number line expands, the upper digits are compressed to fill the remainder of the number line. Given a static length of line with 0 on the left end and 10 on the right end, the lower digits expand towards the right (upper) end of the line. This expansion by the lower digits towards the right end of the number line means that the upper digits are compressed on the right end of the line, in other words, the length between digit 9 and 10 (the right end of the line) would be shorter than the distance between any other digits. These two types of number lines, logarithmic and linear, are displayed in figures 1 through 4.

Certain settings call for a more logarithmic mental representation, or may need no more than a rough, logarithmic approximation, especially when smaller quantities (amounts on the lower end of the number line) are more important than much larger quantities (amounts on the higher end of the number line). As an example, in a survival situation where an individual is stranded in a desert, it may be vital for quantities of water to be estimated in order to conserve water. Large quantities of water are difficult and unnecessary to estimate; if a stranded person has a large quantity of water, he or she may not be in danger of dehydration. Small quantities of water are extremely important and easier to estimate; if the stranded person has a very limited amount of water, he or she may be in danger of dehydration. Likewise, for a hungry animal, the difference between one and two pieces of food is quite important, but the difference between eight and nine pieces of food is negligible.

Numerosity and Number Sense

The term "numerosity" (pp. 35) is used to characterize the fuzzy ability, primarily used by animals and preverbal children, to quantify amounts (Dehaene, 1997). Numerosity has been defined as a rudimentary ability to detect and discriminate non-abstract quantities (Dehaene, 1997). Verbal humans possess a sense of "number" (pp. 35) that is more of an abstract, multi-modal, rigid representation of quantities ordered in a precise sequence of numbers (Dehaene, 1997). There is a developmental continuum of number representation starting with animals and preverbal children to preschool age and elementary school age children to late elementary school age and adulthood. Preschool and early elementary school age children exhibit "numerosity", an initial presentation of an abstract number sense and a logarithmic mental number line. Late elementary school age children have a more developed number sense and their mental number line begins transitioning from logarithmic to linear. Finally, in adulthood a more complete number sense has been acquired and the linear mental number line is dominant.

Many species of animals have been found to possess the ability to discriminate quantities. Tasks involving discrimination are used to test numerosity and number sense. Discrimination is a primary element in the process of estimation. Animals and humans are able to discriminate between both small numbers of objects and large numbers of objects. The estimation task in the current experiment uses small quantities (e.g. 0) and large quantities (e.g. 100) and asks subjects to determine, or discriminate, the value of a digit placed within the small and the large quantities.

Rats and pigeons are two nonverbal animals that have been found to discriminate between two quantities. Researchers have found that rats exhibit numerosity, multimodal manipulation of quantities and absolute numerosity (Davis & Albert, 1986; Meck & Church, 1983, 1984). Meck and Church (1983, 1984) manipulated duration (e.g. short and long) and number (e.g. few and many) of auditory signals to test whether rats could discriminate amongst these criteria simultaneously. The rats discriminated simultaneously between long or short and few or many auditory signals that were associated with two food pellet levers. Meck and Church labeled the rats' ability as being able to "number discriminate", what researchers currently define as "numerosity." The researchers characterized the animals' responses as scalar in nature. A scalar function shares many qualities with the mathematical function used to describe human mental representations: a logarithmic function. It is possible that, by these two function's similarities, the results of Meck and Church's experiments were actually logarithmic.

Davis and Albert (1986) continued Meck and Church's line of experiments on rats by further manipulating quantities to be discriminated. A criticism of Meck and Church's findings was that the ability to discriminate between "few" and "many" quantities may not have been based on a numerical process. Davis and Albert sought to combat this criticism and further prove that rats were, in fact, using a numerical judgment by adding a third quantity. The rats in Davis and Albert's (1986) experiment heard auditory signals of quantities 2, 3 and 4, which determined which food lever to press. Rats were once again able to discriminate among these three quantities, supporting the hypothesis that rats posses a numerical judgment or numerosity.

Roberts and Mitchell (1994) replicated Meck and Church's findings (1983) with pigeons by varying the number and duration of light flashes. In Roberts and Mitchell's experiments, pigeons were able to discriminate between 2 and 8 second flashes, supporting an internal temporal clock. Pigeons were also able to discriminate between 2 and 8 flashes, supporting a mental counting mechanism. Pigeons' counting ability very much supports a theory that they have a sense of "numerosity." Alsop and Honieg's (1991) findings expanded Roberts and Mitchell's findings that pigeons are able to make comparisons of proportions, which further highlights an ability to discriminate quantities (e.g. numerosity).

Monkeys are an example of "verbal" animals (e.g. sign language) that have displayed numerosity in various experimental and naturalistic settings. In 1998, Brannon was able, through training, to teach monkeys Arabic digits and their ordinal place. The monkeys in Brannon's experiments were also able to discriminate among the Arabic digits. This is significant because the Arabic digits in and of themselves are abstract symbols. The monkeys were able to, through trial and error, order digits 1 through 9.

Estimation

Dehaene (1997) has theorized that humans have a mental logarithmic ruler of numbers. This logarithmic ruler as described by Dehaene would inaccurately increase the distance (or magnitude) between numbers on the lower end of a number line as compared to the distance (or magnitude) between numbers on the upper end. An example of this on a 0 to 100 number line would show a subject extending out the 0 to 10 numbers of the line to encompass more length than the 11 to 100 numbers.

Many researchers, such as Dehaene (1997), Geary (2007) and Siegler & Booth (in press) hypothesize that humans inherently use a logarithmic mental number line, cannot inhibit it and have it throughout their lifetime. However, it is extremely likely that, through schooling or instruction, individuals are able to impose a linear mental representation when faced with numerical problems (Dehaene, Izard, Spelke and Pica, 2008). Over the course of three articles and five experiments, Siegler and colleagues showed a progression of numerical mental representations from kindergarten (logarithmic) to adulthood (linear).

Siegler and Opfer (2003) presented second, fourth, sixth graders and college undergraduates with numbers lines of 0 to 100 and 0 to 1,000. The task was chosen by Siegler and Opfer in order to remove any context outside of number relationships and to create a purely numerical task. The authors presented two tasks to subjects: in the first task, place a mark on the number line that corresponds with a number given by the researcher, and in the second task, estimate the numerical value of a mark already placed on the line by the researcher. The numbers chosen for these tasks were over sampled on the lower end of the number spectrum in order to help discriminate between subjects with logarithmic and linear thinking. This initial study provided a foundation for the theory that students of the same age (grade level) use multiple representations (e.g. logarithmic or linear) depending on the stimulus by showing a different pattern of results for the stimulus presentation of 100 and 1,000. The results showed, in subject's accuracy of responses, that very young children (kindergarten and first graders) initially displayed a more logarithmic mental representation, but there was a gradual change to a more linear mental representation throughout school progression, from second grade to adulthood. In fact, a linear model was best fit by the responses of 44 percent of second graders, 56 percent of fourth graders, 81 percent of sixth graders and 100 percent of adult undergraduates.

Another interesting finding of Siegler and Opfer (2003) was the change in mental representations by second graders when faced with different number line denominations. Second graders responses to estimation number lines of 0 to 100 were predominantly linear. However, when the same second grade students were completing 0 to 1,000 number lines, the students responded in a logarithmic fashion. Siegler and Opfer theorized that these students were more versed with digits 0 to 100 and had a more efficient and accurate mental representation of these digits, but second graders are not as exposed to digits 101 to 1,000 and had not created a linear mental representation of this span of digits.

Siegler and Booth expanded Siegler and Opfer's findings in a 2004 experiment displaying number lines beginning with 0 and ending with 100 to kindergartners and first graders. They found support for the theory that children begin school with an "inherent" logarithmic mental number line and acquire a linear mental number line through formal arithmetic education. Again, subjects estimated a digit's location on the number line by placing a hatch mark, with results showing an "inherent" logarithmic number line in kindergarteners' responses to 0 to 100 number lines. In general, the placement of digits 1 through 10 was overestimated; for instance, the digit 10 was placed to the right of the mid point (50) on the 0 to 100 number line. In contrast, the digits 11 through 100 were compressed on the number line, only taking up a very small portion of the right most end of the number line. These results reveal a mental representation of a logarithmic number line, with digits 1 to 10 occupying an inordinate amount of length starting at the left side or beginning (0 point) and reaching into the right half of the number line, and the digits 11 to 100 squeezed into the remainder of the right hand side of the number line ending in 100.

Siegler and Booth performed multiple analyses to detect changes, including: estimation accuracy, response pattern of fit (logarithmic versus linear), variability of responses by individuals and differences across grade. The initial analysis found that students' overall accuracy increased with grade levels from kindergarten to first and then second grade (percent absolute error, respectively= 24%, 14% down to 10%). Response patterns followed a similar trend: logarithmic patterns of responses also decreased with an increase in grade level. Kindergarteners' median responses fit a more logarithmic pattern (R^2=.75), while second graders' median responses fit a more linear pattern (R^2=.97). Variability of subjects' responses also decreased with age/grade level.

As important, Siegler and Booth established a link between improving estimation responses with standardized test scores in mathematics. Significant partial correlations (controlling for age) between accuracy of response (percent absolute error in estimates) and math achievement scores (SAT-9) occurred in all three grades (kindergarten, *pr*(17) =-.32, first grade, *pr*(15)=-.60, second grade, *pr*(17)=-.76). This gave rise to a hypothesis that improving estimation ability (increased accuracy and linearity) may be linked to an increase in overall mathematical ability.

Booth and Siegler (2006) replicated and expanded on their previous findings using number lines of 0 to 100 and 0 to 1,000. The authors once again showed a logarithmic pattern decrease on 0 to 100 number lines from kindergarten (R^2=.92), first grade (R^2=.89), second grade (R^2=.88) to third grade (R^2=.85). The subjects showed an equally striking increase in linear pattern of responses during that same timeframe (R^2=.63, .96, .97, and .98, respectively). In this experiment, the authors once again showed a positive correlation between subject's linearity of response and mathematical achievement scores.

This hypothesis, that all individuals have a logarithmic mental number line and adopt a linear mental representation through schooling, is important for this experiment. Dehaene (1997) and others have claimed that adults still have the original logarithmic mental representation of number, although a linear representation of number has been superimposed on it by formal education. If this is the case, then there should be some task that reveals the operation of the now hidden logarithmic representation. It would probably be difficult to detect the underlying logarithmic representation using accuracy and reaction time data alone. However, using an eyetracker, there may be an initial set of eye movements that suggest reliance on a logarithmic number representation early in processing, possibly followed by eye movements that reflect the operation of the linear number line representation.

Eyetracking

Rayner (1998) gives a massive overview of the current literature, use, technique, findings and applicable theories of eyetracking and eye movements. Rayner states that saccades and fixations are the currently used eye movement data recorded by psychologists in most research involving attention, memory (acquisition and retrieval), reading, and perception. The article highlights the link between eye movement data and real time cognitive processing. Psychologists now use eyetracking methodology and eye movement data to show online cognitive processing. Eye fixation duration may indicate, similar to reaction time, mental processing. Duration and movement data coupled together indicate specific strategy use, as proposed for this experiment.

The eye-mind hypothesis (Just & Carpenter, 1984) is one of the bases for eye movement and eyetracking research. This hypothesis was originally applied to reading tasks and helped explain the number and duration of fixations on words and overall eye movements. The eye-mind hypothesis states that individuals will look at a more difficult concept or construct more often and for longer than easier or more practiced constructs. This hypothesis is based on multiple studies that have shown a strong correlation between the level of difficulty and gaze characteristics.

Haider and Frensch (1999) analyzed eye movements and fixations in order to determine subjects' attending to relevant and irrelevant stimuli during a learning task. The researchers showed that with increased practice, subjects attended to redundant or irrelevant information less, through a decrease in eye fixations on that portion of the stimuli.

Similar to the rationale used by Vigneau et al. (2006) that individuals' eye movements can reveal cognitive processes, this experiment will be recording eye movements to give a more detailed analysis of cognitive processing and strategy use when individuals are performing an estimation task. It is also hoped that through using eye movement data, that we are able to further detail possible overlapping mental representations as hypothesized by Dehaene and by Siegler and associates. Dehaene proposed that individuals initially use a logarithmic mental ruler when faced with estimation tasks and this logarithmic ruler can not be inhibited. A logarithmic mental representation used by adults may be recorded with eye movements shifting more to the higher digits on the number line due to lower numbers occupying greater length. Siegler and associates have also found multiple instances of individuals using different strategies (logarithmic and linear) for different estimation stimuli only moments apart. By recording eye movements of subjects, we may be able to further investigate the characteristics of these strategies, when they are used, and why humans shift from strategy to strategy depending on stimuli.

Math Anxiety

Math anxiety is a condition that arises when individuals are faced with a situation dealing with numbers or arithmetic resulting in apprehension and fear. Math anxiety primarily affects people's ability to perform mathematical tasks, but also has effects on other aspects of individual's lives. Math anxiety may lead individuals to avoid situations that range from simple mathematical tasks to selecting math courses of study. Individuals may become less skilled in math due to avoidance, less practice and performing fewer mathematical problems in academic and real world situations. A meta-analysis of math anxiety research showed a negative correlation between math anxiety and various academic measures: a -.31 correlation between math anxiety and number of high school math classes enrolled and a -.32 correlation between math anxiety and college math courses enrolled (Hembree, 1990). This lack of classroom participation may compound an individual's already poor math performance and increase an individual's math anxiety. Math anxiety is widespread across the population with an estimated 17% of the population classified as being high math anxious (Ashcraft, Krause, & Hopko, 2007).

The first scale used to measure a subject's level of math anxiety was created by Richardson and Suinn in 1972. Richardson and Suinn called their scale the Math Anxiety Rating Scale (MARS), which consisted of 98 items that asked subjects about their feelings involving situations that require mathematics. Participants rated their level of anxiety in various situations using a five point Likert-type scale with results having a reliability of .85 (Brush, 1978). Richardson and Suinn's MARS was shortened from 98 items to 25 questions by Alexander and Martray in 1989 and was titled the shortened Math Anxiety Rating Scale (sMARS). This shortened scale was studied by Fleck, Sloan, Ashcraft, Slane, and Strakowski in 1989 and was correlated with the original MARS at .96.

In the current experiment using the sMARS scale, subjects were divided into three math anxiety categories: low, medium and high. These three categories were determined by using the overall mean (36) and standard deviation (16) of sMARS scores. An individual assigned to the low math anxiety category would be 1 standard deviation below the mean, the medium category subjects had scores that were within .5 standard deviation below and .5 standard deviation above the mean and the high math anxiety individuals were 1 standard deviation above the mean. Demographic information such as the number of high school mathematics courses taken has been found to have a significant correlation with the math anxiety groups (Ashcraft and Kirk, 2001).

In 1992, Eysenck and Calvo proposed the processing efficiency theory that was concerned with the relationship between anxiety and performance. The processing efficiency theory was founded on results that showed higher levels of general anxiety decreased individuals' performance on a secondary task requiring working memory resources. This model was applied to math cognition and math anxiety by Ashcraft and Faust (1994), who found that an individual with math anxiety has competition in working memory between the intrusive thoughts and worry of math anxiety and the actual math task. This competition for limited mental resources will result in longer reaction times and/or inaccuracies (Ashcraft & Faust, 1994).

In Ashcraft and Kirk's (2001) task, subjects were faced with a dual task situation made up of a primary task of addition problems (half with a carry function and half without a carry function) and a secondary task of holding letters in working memory. Subjects were first shown two or six letters that made up the secondary task. The addition problems consisted of two-column addition problem with half the problems involving a carry operation, which are especially important due to the increased working memory requirements for successful completion (LeFevre, DeStafeno, Coleman, and Shanahan, 2005). Subjects were then asked to recall the two or six letters originally displayed. This meant that a subject is maintaining the two or six letters in working memory (particularly the phonological loop) while at the same time completing the addition problem. Maintaining the set of letters in working memory decreases the amount of working memory resources that an individual has to complete the mathematical operations. An individual with high math anxiety would have another decrease in working memory in addition to the decrease due to the secondary letter task.

Ashcraft and Kirk (2001) found that the increase from two to six letter sets and math problems with a carry operation increased error rates. Specifically in experiment 2, which used the situation described above, error rates increased from 4% for low math anxious individuals to greater than 11% for high math anxious individuals completing the 6 letter condition with problems that required carrying (Ashcraft and Kirk, 2001). These results indicate that math anxiety is competing with the letter rehearsal and the math operation for working memory resources.

Estimation may be similar to other math performance areas, in that math anxiety may be correlated with estimation performance. Earlier research has shown that individuals with high levels of math anxiety have poorer overall mathematical skills (Faust, Ashcraft and Fleck, 1996), possibly due to less exposure to numbers and math problems (Hamann & Ashcraft, 1986).

The current experiment seeks to study the effects, if any, of math anxiety on estimation. As in earlier findings, math anxiety may influence reaction time and accuracy on the estimation problems in this experiment, in that subjects may take longer in answering estimation problems and make more and/or greater errors in their answers. Less exposure to arithmetic may also lead to a less concrete "number sense". "Number sense" according to Dehaene (1997) is an intuition of what numbers mean and how they relate to each other. Less exposure to or understanding of numbers and their relationships may lead to miscognitions, such as a logarithmic mental number line as opposed to a more accurate, linear mental number line. A logarithmic mental representation should be detected in subject responses' in this experiment along with eye movement data.

Pilot Experiments

We conducted a number of experiments using a stimulus similar to Siegler's estimation number lines. A number of hypotheses were tested and a few new discoveries about adult's estimation ability were made. An initial goal was to determine the relationship between a paper version of the number line stimulus (as used by Siegler) and a computerized version of the number lines. A new finding from one of our pilot experiments was the accurate and efficient manner in which adults estimate the midpoint of a number line.

We created an experiment that used two types of number lines: one using 0 to 100 endpoints and a second spanning 0 to 1,000. These two number line denominations, 0 to 100 and 0 to 1,000, were presented in both a paper/pencil version and a computerized version. The number lines are a continuous, horizontal line in the center of the piece of paper or center of the computer monitor with 0 slightly below the left end of the line and either 100 or 1,000 slightly below the right end of the line. A vertical hatch mark denoting a numerical value was placed on 26 different points of the number line.

26 different 0 to 100 number lines were created to represent the following values: 3, 4, 6, 8, 10, 12 and 96 on the number line. Mirroring those stimuli, 26 different 0 to 1,000 number lines were created to represent the following values: 31, etc. and 966 on the number line. An over sampling of values at the lower end of the 0 to 100 and 0 to 1,000 spectrums were used to assist in detecting logarithmic responses. In total, data were recorded for 26 paper 0 to 100 and 26 paper 0 to 1,000 number lines and 26 computer 0 to 100 and 26 computer 0 to 1,000 number lines, or 104 total number lines.

The presentation of paper and computerized versions along with the 2 different number line denominations (0 to 100 and 0 to 1,000) were counterbalanced.

Estimation accuracy results from 60 subjects that completed this experiment revealed an extremely high correlation between the two types of stimulus presentation. Errors on 0 to 100 paper number lines were significantly correlated at .911 with 0 to 100 computer number lines. Errors on 0 to 1,000 paper number lines were significantly correlated at .928 with 0 to 1,000 computer number lines. Motivation for establishing the relationship between the paper and computer versions of these estimation number lines were that computer presented stimuli allow reaction times to be recorded and analyzed. Adults' reaction times and accuracy data, when graphed, resemble a vague "M" shape.

This "M" shape highlights an increased accuracy and decreased reaction time when subjects are estimating the 48 and 52 number lines (the midpoint of the number line). Also the "M" shaped accuracy and reaction time results graph show a decreased accuracy and increase in reaction time in the quartile areas (or around the 25 and 75 values on the number line). Accuracy increased again and reaction time decreased around the very ends of the number line (3 at the low end and 96 at the upper end).

These results (the "M" shape) lead to the creation of a number line that would be more difficult for adult subjects to divide in half (e.g. find the midpoint). In the next pilot experiment, paper versions of the stimuli were dropped, and a new computer version of the number line was included: 0 to 723. 723 is a more difficult number to divide in half than 100 or 1,000. As indicated by the previous pilot data, subjects appear to be able to estimate the midpoint of 0 to 100 and 0 to 1,000 number lines more quickly and more accurately than all other points. A midpoint is much more difficult to find and use as a possible reference point on a 0 to 723 number line. Results from this experiment show that subjects were not as accurate in their estimations of any points on the 0 to 723 number lines as they were on any points of the 0 to 1,000 or the 0 to 100 number lines.

In order to investigate differences between individuals that are very accurate estimators and individuals that are poor estimators, we separated subjects by their estimation accuracy for analysis. A group accuracy mean (medium estimator group) was found for subjects' responses on the 0 to 100 number line task along with 1 standard deviation above (good estimator group) and 1 standard deviation below (poor estimator group). Accuracy results of these groups were found to have similarities as well as differences.

All three groups performed similarly on digits that were greater than 50, or the upper half of the number spectrum on the 0 to 100 number lines. Good estimators and medium estimators performed similarly on digits on the lower half of the number line, digits below the 50 digit point. However, poor estimators differed from both good and medium estimators in their estimation accuracy on digits on the lower half of the number line. Inaccuracies confined to the lower half of the number spectrum are characteristic of a more logarithmic number line mental representation. Individuals with a logarithmic number line mental representation would overestimate the placement of the lower half of the numbers on a 0 to 100 number line. Group accuracy differences, along with previous findings, support a hypothesis that an improvement in estimation skills may be due to the decrease in the use of a logarithmic mental number line and the increase in the use of a linear mental number line.

Results of subjects' responses were further analyzed to determine the direction or polarity of inaccuracy. Responses were analyzed on all three denominations: 0 to 100, 0 to 1,000 and 0 to 723. On all three denominations, virtually all subjects' responses were similar in their polarity of error. Responses to nearly all digits that were on the lower end of the number spectrum (e.g., digits less than the midpoint) were overestimates. Responses to nearly all digits that were on the upper end of the number spectrum were underestimates. These two findings, overestimation of lower numbers and underestimation of upper numbers on a number line, are exact characteristics of a logarithmic function. By showing that nearly all subjects' responses were linear, but all the subjects' errors were logarithmic, we have further support that individuals have an inherent logarithmic mental number line that is overlaid with a linear mental number line through formal schooling.

CHAPTER 2

METHOD

Participants

We tested 60 subjects from the UNLV subject pool. The subjects were 18 males (30%) and 42 females (70%) with a mean age of 20.77 and a standard deviation of 6.38. Their year in school broke down as 32 freshmen (53%), 14 sophomores (23%), 5 juniors (8%) and 9 seniors (15%). Ethnically, our subjects were 3 African Americans (5%), 5 Hispanics (8%), 1 Native American (2%), 18 Asian/Pacific Islanders (30%), 24 Caucasian (40%) and 7 identified as other (12%). Of our students who completed this experiment, 43 graduated from Clark County schools (72%) and 17 did not (28%). Demographic information is listed in table 1.

Each subject completed the shortened Math Anxiety Rating Scale (sMARS) to determine their math anxiety level. The sMARS consists of 25 items that ask subjects about their feelings involving situations that require mathematics. The overall mean score on that instrument was 31.85 out of 100 and a standard deviation of 17.81. Math anxiety groups (low, medium and high math anxious) were created using the overall sMARS score of the individual. The math anxiety groups were determined by using the population mean (36) and standard deviation (16). An individual assigned to the low math anxiety category would be 1 standard deviation below the mean (<20 sMARS score), the medium category subjects had scores that were within .5 standard deviation below and .5 standard deviation above the mean (between 26 and 44 sMARS score) and the high math anxiety individuals were 1 standard deviation above the mean (>52 sMARS score). The low math anxious group was made up of 20 subjects (33%), medium math anxious group was made up of 14 subjects (23%), high math anxious group was made up of 11 subjects (18%), and 15 subjects did not fall into any of the three groups (25%).

The Wide Range Achievement Test-3 (Arithmetic) was administered after the sMARS and had an overall mean score of 31.85 out of 40 with a standard deviation of 4.80. Unlike previous experiments, there were no significant correlations between math anxiety (overall score on the sMARS or math anxiety group membership) and math performance (overall score on the WRAT-3). However, a relationship between the number of math classes students have completed and WRAT-3 score was found, $r = .315$, $p < .05$.

Materials

This study used a SensoMotoric Instruments (SMI) Eyelink iViewX Hi-Speed 1250 system. It was a non-invasive, video-based eyetracking system that tracked monocular or binocular eye movements with a sampling rate of 1250 Hz and a tracking resolution of less than .01 degrees. The iViewX eyetracking system was slaved to a Windows based E-Prime program.

Design and Procedure

The primary stimuli used in this experiment were 3 different number lines. The number lines consisted of a continuous horizontal line in the center of the subject monitor with a number "0" below the left end of the line. The number below the right end of the line denoted the line's magnitude. The three magnitudes of the number lines, "100", "723" and "1,000" were randomly presented. Each number line denomination had twenty-six different number lines, each with a different hatch mark across it that denoted a number that the subject estimated and stated out loud. The 100 number line trials had hatch marks corresponding to these numbers: 8, 18, 22, 28, 32, 38, 48, 58, 68, 72, 78, 82 and 88. The 723 number line trials consisted of hatch marks corresponding to these numbers: 58, 130, 159, 202, 231, 275, 347, 419, 492, 521, 564, 593 and 636. The 1,000 number line trials consisted of hatch marks corresponding to these numbers: 88, 188, 228, 288, 328, 388, 488, 588, 688, 728, 788, 828 and 888.

Subjects were then introduced to the SMI eyetracker and the stimulus display. A 13-point calibration procedure was completed to calibrate the eyetracker to the subject's eye movements. Subjects were then shown practice instructions and for each estimation number line denomination (e.g. 0 to 100, 0 to 1,000 and 0 to 723) subjects completed two practice trials. Instructions for the actual task followed along with the actual task of seventy-eight number lines.

The practice instructions were: "In this task you will be shown three types of estimation lines with end points of 0 to 100, 0 to 723, and 0 to 1,000. Each line had a perpendicular mark that denotes a number on that line. Your task is to say out loud what number you think corresponds to that mark. Please try not to make any noises other than your answer. You will be shown practice trials for each of the number line denominations. Do you have any questions?" These instructions were followed by six practice trials and then instructions for the actual task: "As in the practice trials in this task you will be shown three types of estimation lines with end points of 0 to 100, 0 to

723, and 0 to 1,000. This line will have a perpendicular mark that denotes a number on that line. Your task is to say out loud what number you think corresponds to that mark. Please try not to make any noises other than your answer. Do you have any questions?" After the instructions were given, the experimenter prompted E-prime to begin the experiment by pressing a button on the keyboard, and a ready prompt, consisting of the word "Ready", appeared in the center of the screen for 1 second. The number line stimulus was immediately displayed and consisted of a continuous horizontal line in the center of the subject monitor with a number "0" below the left end of the line. The number below the right end, denoting the line's magnitude, consisted of "100", "723" and "1,000" and was randomly presented. At the conclusion of the estimation number line task, the subject was asked: "Did you use a midpoint of the number line when estimating the value of the hatch mark? What did you use, if anything, as the numerical midpoint of the number lines with endpoints of 723?"

CHAPTER 3

RESULTS

This experiment recorded four different, dependent variables: subjects' response, reaction time, number and location of eye fixations and the duration of each fixation. Two variables were derived from the subjects' responses: relative errors and directional errors. Subjects' responses consisted of numerical values that they estimated to be the value of the mark on each estimation line and were stated out loud. Reaction times were how quickly subjects voiced their responses to the experimental stimuli. Eye fixations and their duration were recorded during the presentation of each estimation line, ended by the subject giving their response out loud.

Using subjects' responses, we were able to calculate both relative and directional errors. Relative errors were computed from the absolute difference between the actual value of the mark on the estimation line and the subjects' response. For example, if a subject's response to an estimation line with a mark at the 50 digit point was 40, then the relative error would be 10, due to the difference between the response and actual value of the mark. Relative errors show how well or poorly individuals estimate the value of the mark on each number line. Relative errors were used instead of absolute errors in order to allow for direct comparison between the results of the three different denominations and standardization to the 0 to 100 number line results.

Directional errors are calculated similarly to relative errors, but the formula does not include finding the absolute difference. For example, if a subject's response to an estimation line with a mark at the 50 digit point was 40, then the directional error would be -10 or an underestimate by 10 digits. Directional errors reveal whether responses were below or above the value of the mark. A positive directional error would mean that the subject's response was above the value of the mark, in other words it was an overestimate. Conversely, a negative directional error would mean the response was less than the value of the mark, or an underestimate.

Relative errors would also show which digit is more difficult to estimate than the others and directional errors would show which variable may consistently be over- or under- estimated.

A total of 60 subjects participated in this experiment of 26 estimation lines presented within each of 6 blocks of trials for a grand total of 9,360 trials. Upon completion of data collection and prior to data analysis, three types of data points were removed: microphone errors, outliers and wrong denomination responses. Microphone errors consisted of trials in which subjects did not respond loudly enough during the experiment. Reaction times for these types of trials were removed from analysis. Outlier relative errors and reaction times were detected by using the Dixon's outlier test (Verma & Quiroz-Ruiz, 2006). Trials were also removed if the subject's response did not readily fit within the range of the denomination of that estimation line. Here are the guidelines for a response that would be considered a wrong denomination response: on a 0 to 100 number line, any response above 100 would be removed; on the 0 to 1,000 number lines, responses that were primarily within the 0 to 100 range were removed (In other words, if the participant mistakenly assumed the number line went only to 100 and responded as such for an appreciable number of these trials). Subjects' responses, relative errors, directional errors and reaction times were all removed for a trial in which an outlier was detected or responses were in the wrong denomination. Eye data (i.e., fixations and dwell times) were also removed for trials that either had microphone errors, had an outlier or wrong denomination response. In the 723 denomination, two estimation lines were removed from analysis due to data corruption: 130 and 636. This amounted to total 360 trials, four within block C, one within block E and one within block F, for each subject were removed. A grand total of 1,363 trials or 14.56 percent of the 9,360 total trials were removed from analysis. Fully 620 of these trials were due to microphone errors (45.5%) and only 140 (10.3%) were outliers. Of the discarded trials due to denomination errors, 226 occurred in block D, the first block that mixed two denominations together. For a complete breakdown of removed trials per block, see table 2.

Linearity

Siegler and Opfer (2003) found that adults' responses to simple 0 to 100 and 0 to 1,000 number lines, such as the ones used in this experiment, were overwhelmingly linear. An example of linear responses to this experiment is shown in figure 8. A curve estimation analysis of the 60 subjects in this experiment (table 3) found similar results to previous adult number line estimation experiment results. The overall correlation between a linear function and all 60 subjects' responses was .971 on the block A (100) number lines. On the block B (1000) number lines, subjects' responses correlated with a linear function at .966, as well. Subjects' responses for block C (723) number lines correlated with a linear function at .937. These correlations support a theory (Siegler and

Opfer, 2003) that adults' possess a linear mental number line and also show a slight increase in difficulty from the 0 to 100 to the 0 to 1,000 to the 0 to 723 number lines.

Relative Errors (overall)

In an effort to compare results between denominations of estimation lines, "relative" errors were calculated instead of "absolute" errors. Absolute errors and relative errors share a common quality: they both show the difference between the actual value of the mark on the estimation line and the subject's response. However, absolute errors do not take into account, and thereby make it difficult to compare, errors across number line denominations. For example, if a subject responds 20 on a 0 to 100 number line with a mark at the 10 digit, the absolute difference would be 10; whereas if a subject responds 200 on a 0 to 1,000 number line with a mark at the 100 digit, the absolute error would be 100. It would be difficult to compare the absolute error for a 0 to 100 number line with a commensurate 0 to 1,000 number line (or a 0 to 723 number line). Relative errors, however, take into account the increases in denomination from 100 to 723 or 1,000. By dividing the actual difference between the value of the mark and the subject's response by the denomination of the estimation line, the results will be comparable to the 0 to 100 results.

For example, a subject completing a 0 to 100 number line with a mark corresponding to 48 and responded that the value of the mark was 50, the relative error for that number line would be 2. Relative errors for 0 to 1,000 number lines were determined using a slightly different equation: the difference between the correct value of the mark on the number line and the subject's response was divided by 10. An example of a relative error on a 0 to 1,000 number line would be: if the mark equaled

480 and the subject responded 500, the difference would be 20, but the relative error would be 20 divided by 10, or 2.0. Relative errors for 0 to 723 number lines are found in the same manner, but the divisor is 7.23 instead of the 10 used for 0 to 1,000 number lines. This procedure converts errors to percentages, allowing for the direct comparison between the results of the three different denominations and standardization to the 0 to 100 number line results.

The relative errors for each block can reveal difficulty level of each number line denomination. The increase in errors from block A to block B and finally block C are readily apparent in table 4 and give support to a theory of increased difficulty among the number line denominations (Siegler & Opfer, 2003), with 0 to 100 being the easiest and 0 to 723 being the most difficult. Blocks D, E and F are slightly different from block A, B and C, in that they are not made up of a single denomination of number lines. Block D consists of equal number of trials with 0 to 100 and 0 to 1,000 number line, block E: 0 to 723 and 0 to 1,000 and block F: 0 to 100 and 0 to 723. Further investigation beyond just the overall block means of these mixed blocks of number lines into the separate denominations reveals a difference in error rates from the earlier, pure blocks of A, B and C. A comparison of the pure and mixed block relative error, directional error and reaction time mean results is found in table 5.

By graphing relative errors against digits to be estimated, a noticeable "M" shape appeared. This "M" shape highlights an increased accuracy when subjects are estimating the value of a 0 to 100 number line with a mark at the 48 (roughly the midpoint of the number line). Also the "M" shaped relative error graph shows a decreased accuracy in the quartile areas (or around the 22-28 and 72-78 values on a 0 to 100 number line).

Accuracy increased again around the very ends of the number line (8 at the low end and 88 at the upper end). This trend, ease at estimating the midpoint and difficulty at determining the quartile areas, was found for nearly all subjects in varying degrees in all three of our denominations of number lines (100, 723 and 1,000). This pattern replicates obtained in our pilot experiments.

Figure 9, panels A, B and C show the "M" shaped relative error results on the 100 number lines, both pure and mixed blocks. All three figures show the high accuracy that individuals estimate the midpoint and the poor accuracy at estimating the quartiles (e.g., digits 22-32 and digits 68-82).

Figure 10, panels A, B and C again show the "M" shaped relative error results on the 1,000 number lines, both pure and mixed blocks. All three figures show the high accuracy that individuals estimate the midpoint and the poor accuracy at estimating the quartiles (e.g., digits 228-328 and digits 688-828).

Figure 11, panel A, B and C again show the "M" shaped relative error results on the 723 number lines although with higher overall means. All three figures show the high accuracy that individuals estimate the midpoint and the poor accuracy at estimating the quartiles (e.g., digits 159-275 and digits 419-564).

Reaction Time Results (overall)

Similar to relative error results, reaction time results form an "M" shape when graphed by digits on the number lines. This "M" shape highlights a decreased reaction time when subjects are estimating the 48 and 52 number lines (the midpoint of the 0 to 100 number lines). Also the "M" shaped reaction time figures show an increase in reaction time in the quartile areas (or around the 22-28 and 72-78 values on the 0 to 100 number lines). Reaction times decreased around the very ends of the number line (8 at the low end and 88 at the upper end). These reaction time results overlaid with the relative error results fully show the ease with which subjects estimate the midpoint and their difficulty at determining the quartile areas. Again this pattern of reaction times was found for nearly all subjects in all three of our denominations of number lines (100, 723 and 1,000).

Figure 12, panels A, B and C show the "M" shaped reaction time results of the 100 estimation lines. The midpoint was extremely accurately estimated in the relative error results and the reaction time results show just how quickly this was accomplished. However, the reaction time results show that individuals spent longer estimating the quartiles, but this did not seem to help in accuracy.

Figure 13, panels A, B and C shows the same results for the 1,000 estimation lines as the 100 estimation lines, in that, subjects very rapidly estimated the values of the midpoint, but were much slower in the quartiles.

Figure 14, panels A, B and C show the reaction time results for the 723 estimation lines, both pure and mixed blocks. The pure block reaction times results in figure 14 show the difficulty that individuals have with estimating the midpoint of 723 by the lack of dip in the reaction time at the midpoint (this was the first encounter with this denomination for many subjects). Subjects appear to have taken longer, in order to possibly perform the calculation, followed by the verbalized estimate.

Directional Errors (overall)

In addition to relative errors, directional errors were calculated in order to determine if subjects overestimated or underestimated the value of the certain digits.

Underestimates, or estimates that were less than the actual value of the mark on the number line shown, were considered negative errors. Overestimates are estimates that were greater than the actual value of the mark on the number lines shown and were considered positive errors. A trend in overestimates or underestimates may reveal aspects of subject's mental number line or cognitive processes.

Table 5 shows mean relative errors, directional errors and reaction times for each denomination of number lines, separated for each block of trials, and also the results of the simple ANOVA's which contrasted the error rates and reaction times between the lower and upper halves (spectrums) for the number lines (e.g., errors from 0 to 50 and 51 to 100 on the 100 number lines). Subjects' directional error results were found to be extremely similar for 100 and 1,000 number lines as shown in figures 15 and 16 . These directional error figures have three major characteristics: a horizontal, "zero" line that divides positive from negative errors, the length of the error line that is above the "zero" line (i.e., positive errors) and the length of the error line that is below the "zero" line (negative errors). Subjects consistently overestimated all digits below 50 on the 100 number lines and all digits below 500 on the 1,000 number lines. The opposite, subjects' responses were overestimates, occurred for all digits above 50 on the 100 and above 500 on the 1,000 estimations lines.

For example, figure 15, panels A, B and C, all 100 number line directional error results, show that responses for digits 8 to 58 were all overestimates and above the "zero" line. In these same figures, all responses to digits 58 to 88 were underestimates and below the "zero" line. Similarly, 1,000 number line results in figure 16, panel A show responses to digits 88 to 388 were all overestimates and above the "zero" line and responses to digits 488 to 888 were all underestimates and below the "zero" line. Figure 15, panels B and C are 1,000 number line directional error results as well and show similar, although not the exact same results.

Directional errors on 723 number lines were quite different than errors on 100 and 1,000 number lines, in that the errors were all overestimates with no transition from positive to negative errors (or over- to underestimates). Further investigation has revealed a decided difference between error polarity in the lower end of the number line (i.e., 0 to 50) and the upper end of the number line (i.e., 50 to 100). While figures 15 and 16 show a difference between errors on the two ends of the number spectrum, we tested whether it was statistically significant or not. Again, this was investigated because individuals should perform consistently throughout the number line; there should be no difference between estimating the values of digits below the midpoint or above the midpoint. Table 6 shows that on the 723 and 1,000 number lines, despite the fact that subjects did not make different relative errors on the lower compared to upper number spectrums, their errors were significantly different in polarity.

Table 6 details the relative and directional errors and reaction times, and divides each by the lower and upper spectrum of numbers for each block of trials. For block A (100), relative errors were significantly less for the lower spectrum (M = 4.57, SE .143) than the upper spectrum (M = 4.06, SE .125), $F(1, 1421)$_ = 7.113, $p<.001$, directional errors were significantly greater (i.e. positive errors or overestimates) for the lower spectrum (M = 3.60, SE = .178) than the upper spectrum (M = -1.36, SE .191), $F(1, 1421) = 361.293$, $p<.0005$, but reaction times did not differ between the spectrums. Block B (1000) only had a significant difference in directional errors ($F(1, 1404)$ =

245.593, $p<.0005$), with subjects having a more positive errors for the lower spectrum ($M = 2.27$, $SE = .218$) than the upper spectrum ($M = -2.66$, $SE = .227$). Block C (723) also only had a significant difference in directional errors ($F(1, 1192) = 6.527$, $p<.05$), but both spectrums had positive errors (Lower: $M = 5.02$, $SE = .341$, Upper: $M = 3.78$, $SE = .344$). These trends in results for the 100, 1000 and 723 denominations were repeated for the mixed blocks as well.

Math Anxiety Groups' Relative Errors

Subjects were divided into groups using their level of math anxiety. Using population mean and standard deviation as detailed earlier, three math anxiety groups were created: low, medium and high math anxious. In simple arithmetic tasks, such as single digit addition, a math anxiety effect has not been found. However, these effects have been found when the difficulty of the task begins to strain working memory capacity (e.g., Ashcraft and Kirk, 2001) and thereby task completion competes with math anxiety for those limited resources. Estimation is thought to be a rudimentary task that is below a simple arithmetic task in difficulty and should not require a great amount of working memory resources. Due to this reasoning, math anxiety should not have a significant effect on estimation performance. This was found for 100 and 1,000 estimation line results, but surprisingly performance on 723 lines had significant math anxiety effects. All three estimation groups performed similarly on 100 estimation lines and 1,000 estimation lines (figures 18 and 19, respectively). But, figure 20, panels A, B and C show the separation between high math anxious individuals and all others. This may be due to the increased difficulty of the 723 estimation lines that brought about an increased competition with math anxiety for limited working memory resources.

A one-way analysis of variance (ANOVA) on the relationship between math anxiety group membership and relative errors per block was performed. The math anxiety groups' relative error results from block A (100) had an overall significant difference, $F(2,1154) = 5.177, p<.001$, but once each group was compared post hoc, using Bonferroni's t statistic (used for all post hoc testing), there was only a significant difference between the medium group ($M = 3.71, SE = .143$) and the high group ($M = 4.63, SE = .207$), $p<.005$. Block C's (723) results yielded a number of significant differences, including an overall difference between math anxiety group and relative error, $F(2,983) = 30.928, p<.0005$. All three math anxiety groups' relative errors were significantly different: medium ($M = 5.27, SE = .249$) math anxious individuals had significantly lower relative errors than both low ($M = 6.70, SE = .293$), $p<.05$, and high ($M = 9.29, SE = 4.72$), $p<.0005$, math anxious individuals, but low math anxious still had significantly lower relative errors than high, $p<.0005$.

Block E was a mixture of two different denominations, 723 and 1,000. An analysis of the two denominations together found an overall significant difference between the math anxiety groups' relative errors, $F(2,1070) = 16.906, p<.0005$ and a difference between the medium ($M = 4.27, SE = .213$) and high ($M = 6.67, SE = .399$) math anxious groups, $p<.001$. A similar pattern of results was found for the 723 estimation lines in this block as in the pure 723 block: an overall significant difference between the math anxious groups, $F(2,485) = 20.133, p<.0005$. The high ($M = 9.09, SE = .655$) math anxious had higher relative errors than both low ($M = 6.18, SE = .333$) and medium ($M = 4.98, SE = .353$) math anxious individuals, $p<.01$ and $p<.0005$ respectively.

Block F was a mixture of both 100 and 723 estimation lines. Again, an overall significant difference between the math anxiety groups for the combined denomination was found, $F(2, 997) = 7.757, p<.0005$. The medium ($M = 4.78, SE = .375$) math anxious subjects had significantly lower relative errors than the high ($M = 6.56, SE = .365$) math anxious subjects, $p<.05$. When the 100 number line results are removed from the analysis, another overall significant difference between the three math anxious groups is found, $F(2,442) = 8.901, p<.0005$. Specifically, the medium ($M = 5.13, SE = .585$) group had significantly lower relative errors than the high ($M = 8.15, SE = .646$) math anxious, $p<.0005$.

A 3 (Math Anxiety Group: Low, Medium and High) x 2 (Denomination: 100 and 723) x 2 (Spectrum: Lower and Upper) mixed design ANOVA was performed to further investigate the relationship between math anxiety, estimation line denomination and number spectrum on the estimation lines. Relative error was the dependent variable, math anxiety group was treated as the between subjects variable and denomination and spectrum were within-subjects variables. Significant main effects were found for math anxiety group ($F(2, 42) = 4.422, MSE = 15.426, p<.05, \eta_p^2 = .174$) and for spectrum ($F(1, 42) = 41.491, MSE = 5.739, p<.0005, \eta_p^2 = .498$). Figures 21 and 22 show that when comparing medium with high math anxiety group, relative error increased, ($M = 4.694, SE = .525$ and $M = 7.027, SE = .592$, respectively). Figures 21 and 22 also show the significant main effect in relative error between the two number spectrums, such that relative errors increased from the lower spectrum ($M = 4.550, SE = .256$) to the upper spectrum ($M = 6.923, SE = .429$). There was also a two-way interaction found between spectrum and math anxiety group ($F(2, 42) = 3.884, MSE = 5.739, p<.05, \eta_p^2 = .156$).

Math anxiety groups had fewer relative errors on the lower spectrum (low: $M = 4.429$, $SE = .373$, medium: $M = 4.113$, $SE = .445$ and high: $M = 5.109$, $SE = .503$) than on the upper spectrum (low: $M = 6.550$, $SE = .625$, medium: $M = 5.275$, $SE = .747$ and high: $M = 8.945$, $SE = .842$), but the high math anxiety group increased more than the low or the medium in the upper spectrum. A two-way interaction was also found between spectrum and denomination ($F(1, 42) = 4.906$, $MSE = 2.582$, $p<.05$, $\eta_p^2 = .105$). There was a greater decrease in relative errors for the upper spectrum as the denomination increased from 100 (lower: $M = 5.086$, $SE = .369$ and upper: $M = 6.913$, $SE = .559$) to 723 (lower: $M = 4.014$, $SE = .225$ and upper: $M = 6.934$, $SE = .433$).

Math Anxiety Groups' Reaction Time

Reaction time provides the second aspect of the midpoint strategy: individuals estimate the midpoint very rapidly, but take much longer to estimate the quartiles. This is again occurring on the 100 and 1,000 number lines when we divide the reaction time results by our math anxiety groups. The quick determination of the midpoint is very visible in the 1,000 number lines, as shown in figure 24, panels A, B and C, but still discernable on the 100 number line results in figure 23, panels A, B and C. The 723 number line results show a separation of groups, but this time it is the low anxiety group that separates itself: these individuals take much longer to estimate almost all digits. In fact, for almost all estimation line denominations, low math anxious individuals have longer reaction times than medium or high math anxious subjects. This group of results would parallel earlier results from the 723 estimation lines and previous research that deals with high math anxious individuals completing tasks very rapidly to escape the current math task.

A one-way analysis of variance (ANOVA) on the relation of math anxiety group membership and reaction time per block was performed. The math anxiety groups' reaction time results from block A (100) had an overall significant difference, $F(2,1139) = 7.096$, $p<.001$, but once each group was compared post hoc, using Bonferroni's t statistic (used for all post hoc testing), there was only a significant difference between the low group ($M = 3178$, $SE = 92$) and the high group ($M = 2644$, $SE = 93$), $p<.0005$ and $p<.0005$ respectively.

Block B's (1,000) results showed a significant difference in reaction times between math anxiety groups, $F(2,1117)=6.521$, $p<.005$. The low group ($M = 3607$, $SE = 117$) had much longer reaction times than both the medium group ($M = 3175$, $SE = 113$) and the high group ($M = 3007$, $SE = 133$), $p<.05$ and $p<.0005$ respectively.

Block C's (723) results yielded a number of significant differences, including an overall difference between math anxiety group and relative error, $F(2, 971) = 20.880$, $p<.0005$. Low math anxious individuals ($M = 4783$, $SE = 189$) had much longer reaction times than medium ($M = 3801$, $SE = 161$) or high math anxious individuals ($M = 3218$, $SE = 105$), $p<.0005$. Medium math anxious individuals also had longer reaction times than high math anxious subjects, $p<.005$.

Block D was a mixture of two different denominations, 100 and 1,000. Overall, math anxiety groups had significantly different reaction times when completing this block, $F(2, 945) = 20.191$, $p<.0005$. Keeping both denominations together, math anxious groups differentiated themselves, with the low math anxious individuals ($M = 3825$, $SE = 122$) having much higher reaction times than either medium ($M = 3003$, $SE = 105$) or high ($M = 2920$, $SE = 86$) math anxious individuals, $p<.0005$ and $p<.0005$ respectively.

Focusing on the 100 number lines, the three groups continued to have different reaction time lengths, $F(2,516) = 9.810, p<.0005$. Low math anxious subjects ($M = 3619$, $SE = 152$) continued to have longer reaction times than medium ($M = 2806$, $SE = 131$) or high ($M = 2971$, $SE = 121$) math anxious subjects, $p<.0005$ and $p<.0005$ respectively. A significant difference in reaction times was also found between the three math anxiety groups in the 1000 number lines of block D, $F(2,428) = 11.142, p<.0005$. The low math anxious group ($M = 4060$, $SE = 195$) again had longer reaction times than either the medium ($M = 3239$, $SE = 169$) or high ($M = 2850$, $SE = 110$) math anxiety groups, $p<.001$ and $p<.0005$ respectively.

Block E was a mixture of both 723 and 1,000 estimation lines. Keeping these two denominations together, a significant difference in reaction times was found, $F(2,1012) = 8.713, p<.0005$. High math anxious subjects ($M = 3383$, $SE = 113$) had significantly faster reaction times than both low ($M = 4296$, $SE = 146$) and medium ($M = 3947$, $SE = 166$) math anxious subjects, $p<.0005$ and $p<.05$ respectively. Analyzing the relationship between the three math anxiety groups on the 723 number lines alone revealed a significant difference, $F(2,457) =9.021, p<.0005$. A significant difference in reaction times was found in the 723 estimation line results, such that low math anxious subjects ($M = 5149$, $SE = 270$) were slower than high ($M = 3542$, $SE =154$) math anxious subjects, $p<.0005$. Medium math anxious subjects were also significantly slower than high math anxious subjects, $p<.0005$. The results of the 1,000 estimation lines in block E did not yield a significant difference in reaction times between all three math anxiety groups.

Block F was a mixture of both 100 and 723 estimation lines. Again, an overall significant difference between the math anxiety groups was found for the combined denominations, $F(2,924) = 3.718, p<.05$. Separately, there was no difference between the two denominations and all three math anxiety groups.

A 3 (Math Anxiety Group: Low, Medium and High) x 2 (Denomination: 100 and 723) x 2 (Spectrum: Lower and Upper) mixed design ANOVA was performed to further investigate the relationship between math anxiety, estimation line denomination and number spectrum on the estimation lines. Reaction time was the dependent variable, math anxiety group was treated as the between subjects variable and denomination and spectrum were within-subjects variables. A significant main effect was found for denomination ($F(1, 42) = 21.909$, $MSE = 2265538.524$, $p<.0005$, $\eta_p^2 = .343$). In comparing figure 26 with figure 27, the significant increase in reaction time from 100 denomination ($M = 3112.551$, $SE = 218.606$) to 723 denomination ($M = 4194.716$, $SE = 410.735$) is visible. A two-way interaction between denomination and spectrum ($F(1, 42) = 6.010$, $MSE = 148456.581$, $p<.05$, $\eta_p^2 = .125$) was also found. In the 100 denomination reaction times decreased from the lower ($M = 3203.212$, $SE = 216.688$) to the upper ($M = 3021.891$, $SE = 226.593$) spectrum, but in the 723 denomination reaction times increased from the lower ($M = 4140.290$, $SE = 383.850$) to the upper ($M = 4249.142$, $SE = 445.382$) spectrum. No other significant interactions were found.

Math Anxiety Groups' Directional Errors

Similar to the math anxiety groups' relative error results, math anxiety did not have a significant effect on directional errors on the 100 and 1,000 estimation lines. Low, medium and high math anxious subjects overestimated digits in the lower end of the number spectrum and underestimated digits in the upper end of the number spectrum on both 100 and 1,000 number lines as shown in figures 28 and 29, panels A, B and C for each. These figures mirror the over- then underestimate trends found in the overall directional error trends found in figures 15 and 16panels A, B and C for both. However, figure 30 panels A, B and C show that high math anxious individuals did distinguish themselves on the 723 estimation lines. These figures show that low and medium math anxious subjects overestimated nearly all digits on the 723 estimation lines, but not nearly to the magnitude of the high math anxious subjects.

A one-way analysis of variance (ANOVA) on the relation of math anxiety group membership and directional errors per block was performed. The math anxiety groups' directional error results from block A (100) had an overall significant difference, $F(2, 1156) = 11.155$, $p<.0005$. Groups were compared using Bonferroni's t statistic (used for all post hoc testing). There was a significantly lower directional error mean between both the low math anxious group ($M = .69$, $SE = .252$) and the medium group ($M = .63$, $SE = .241$) and the high group ($M = 2.38$, $SE = .315$), $p<.0005$ for both.

Block B's (1,000) results showed no significant difference in directional errors between the math anxiety groups.

Block C's (723) results yielded a number of significant differences, including an overall difference between math anxiety group and mean directional error, $F(2, 983) = 33.555$, $p<.0005$. Low ($M = 3.34$, $SE = .405$) and medium ($M = 2.17$, $SE = .371$) math anxious individuals had significantly lower mean directional errors compared to high math anxious individuals ($M = 7.60$, $SE = .584$), $p<.0005$.

Block D was a mixture of two different denominations, 100 and 1,000. Overall, math anxiety groups had significantly different directional error means when completing this block, $F(2, 1004) = 7.618$, $p<.001$. Keeping both denominations together, math anxious groups differentiated themselves, with both low ($M = .09$, $SE = 2.66$) and medium ($M = -.09$, $SE = 2.88$) math anxious individuals having significantly lower directional error means than high ($M = 1.61$, $SE = .383$) math anxious individuals, $p<.005$ and $p<.001$. A significant difference in directional error means was also found between the three math anxiety groups in the 1000 number lines of block D, $F(2,457) = 6.581$, $p<.005$. The low ($M = -.94$, $SE = .385$) and medium ($M = -1.27$, $SE = .399$) math anxious groups had significantly lower directional errors compared to the high ($M = 1.12$, $SE = .599$) math anxiety group, $p<.005$ for both.

Block E was a mixture of both 723 and 1,000 estimation lines. Keeping theses two denominations together, a significant difference in directional error means was found, $F(2,1070) = 22.059$, $p<.0005$. High math anxious subjects ($M = 3.88$, $SE = .521$) had significantly higher directional errors than both low ($M = .58$, $SE = .325$) and medium ($M = .49$, $SE = .316$) math anxious subjects, $p<.0005$ for both. Analyzing the relationship between the three math anxiety groups on the 723 number lines alone find a significant difference, $F(2,485) =21.132$, $p<.0005$. Low ($M = 2.39$, $SE = 5.12$) and medium ($M = 1.89$, $SE = .517$) math anxious groups had significantly lower directional error means than the high ($M = 7.33$, $SE = .818$) math anxious subjects, $p<.0005$ for both. Medium math anxious subjects were also significantly slower than high math anxious subjects, $p<.0005$. The results of the 1,000 estimation lines in block E found a significant difference in directional error means between the three math anxiety groups,

$F(2,584) = 4.702, p<.01$. Specifically, the low math anxious group ($M = -.92$, $SE = .393$) had a significantly lower directional error mean than the high math anxious group ($M = .96$, $SE = .563$), $p<.01$.

Block F was a mixture of both 100 and 723 estimation lines. Again, an overall significant difference between the math anxiety groups was found for the combined denominations, $F(2,997) = 14.492, p<.0005$. Both the low ($M = 1.79$, $SE = .333$) and medium ($M = 1.39$, $SE = .457$) math anxious directional error group means were significantly lower than the high math anxious group mean ($M = 4.52$, $SE = .473$), $p<.0005$ for both. A significant difference between the three math anxiety groups was found for the 723 number lines within block F, $F(2,442) = 14.528, p<.0005$. Again, both low ($M = 2.05$, $SE = .505$) and medium ($M = .96$, $SE = .727$) math anxious individuals had significantly lower directional error means than the high ($M = 6.18$, $SE = .822$) math anxious individuals, $p<.0005$ for both.

A 3 (Math Anxiety Group: Low, Medium and High) x 2 (Denomination: 100 and 723) x 2 (Spectrum: Lower and Upper) mixed design ANOVA was performed to further investigate the relationship between math anxiety, estimation line denomination and number spectrum on the estimation lines. Directional error was the dependent variable, math anxiety group was treated as the between subjects variable and denomination and spectrum were within-subjects variables. Significant main effects were found for math anxiety group ($F(1, 42) = 6.529$, $MSE = 24.075, p<.005$, $\eta_p^2 = .237$), denomination ($F(1, 42) = 17.978$, $MSE = 13.600, p<.0005$, $\eta_p^2 = .300$) and for spectrum ($F(1, 42) = 28.220$, $MSE = 475.369, p<.0005$, $\eta_p^2 = .402$). Figures 31 and 32 show that as math anxiety group changed from medium to low to high, directional error increased or became more positive ($M = 1.503$, $SE = .656$, $M = 2.019$, $SE = .549$, $M = 4.838$, $SE = .740$, respectively). A significant interaction between denomination by spectrum was also found ($F(1, 42) = 158.301$, $MSE = 6.072$, p<.0005, $\eta_p^2 = .383$) with a decrease in directional error from the lower spectrum to the upper spectrum for both denominations, but much more so in the 100 denomination (Lower: $M = 4.226$, $SE = .462$, Upper: $M = -1.055$, $SE = .420$) than the 723 denomination (Lower: $M = 4.695$, $SE = .774$, Upper: $M = 3.279$, $SE = .663$). No other significant interactions were found.

Estimation Groups' Relative Errors

As in most arithmetic tasks, there are good performers and not so good performers; the estimation task should be, and was, no different. It was hypothesized that poor estimators would differentiate themselves on certain aspects of the estimation tasks, such as poorer performance in the upper end of the spectrum as opposed to the lower end of the spectrum. In order to investigate possible differences between proficient estimators and poor estimators, subjects were separated into three estimator groups by their relative error score on the 723 estimation lines. The 723 estimation line relative error results in block C were used because of the previous findings that these estimation lines had a greater variance and therefore might be an instrument that would differentiate estimation ability. Block C's (723) overall relative error mean and standard deviation were found by calculating each subjects' mean for all estimation lines in that block and then determining the mean of all of the subject's means. Using block C's overall mean and standard deviation, three groups of estimators were created: Poor estimators, average estimators and good estimators. Poor estimators were individuals who had a mean relative error greater than one half a standard deviation above the overall mean. The average estimators had a mean relative error between one half standard deviation below to one half standard deviation above the overall block mean relative error. Good estimators had a mean relative error less than one half standard deviation below the overall relative error mean of that block.

Estimator groups and math anxiety groups had very similar membership. There were a total of 20 low math anxious individuals in this experiment and a total of 26 subjects that were considered good estimators with 11 subjects that were shared. The medium math anxiety group was made up of 14 subjects with 6 of those being shared with the average estimators. The high math anxiety group was made up of 11 subjects, 6 of those were also in the poor estimator group.

Block C had an overall mean relative error of 7.81 and a standard error of .484. Using the above formula, there were 15 subjects that were poor estimators, with a group relative error mean and standard error of 12.26 and .701 respectively. The average estimator group was made up of 19 subjects with a mean relative error of 6.89 and standard error of .239. The good estimator group was made up of 26 subjects with a mean relative error and standard error of 4.26 and .220. Poor estimators did worse than even the average estimators by a wide margin as shown in table 16. Figure 33. panel C shows that the good and average estimators were able to once again find the midpoint accurately, while the poor estimators could not. As shown in table 17, good estimators had a significantly lower relative error mean than either average or poor estimators, and average estimators had a lower relative error mean than the poor estimators ($p<.0005$ for each).

Block A (100) had an overall relative error mean of 4.29 and a standard error of .162. Using the estimator groups' membership from block C (723), the estimator groups' relative error means were significantly different ($F(2, 77) = 12.207$, $MSE =$, $p<.0005$) as poor estimators ($M = 5.25$, $SE = .340$) had significantly higher relative errors than average ($M = 4.02$, $SE = .216$) or good ($M = 3.58$, $SE = .145$) estimators as shown in table 16. Table 17 shows that good estimators and average estimators had significantly lower relative error means than poor estimators ($p<.0005$ and $p<.005$, respectively).

Block B (1000) had an overall relative error mean of 4.66 and a standard error of .191. Applying the estimator group membership to the block B results, there was a significant difference in relative errors ($F(2, 77) = 13.524$, $MSE =$, $p<.0005$) which was characterized by good estimators ($M = 3.48$, $SE = .197$) having lower relative errors than average ($M = 5.51$, $SE = .317$) and poor ($M = 4.99$, $SE = .328$) estimators as table 16 shows. Table 17 shows that good estimators had significantly lower relative errors than average and poor estimators ($p<.0005$ and $p<.001$, respectively).

Unlike the relative error results of block C (723) shown in figure 33, panel C, all three estimator groups' relative error results for both block A (figure 33, panel A) and B (figure 33, panel B) show the midpoint strategy in that subjects were able to accurately estimate the midpoint of the estimation line and had more difficulty in finding the quartile errors.

Due to the method used to create the estimator groups, it is a given that their relative errors would be significantly different. However, in pilot experiments estimator groups were primarily distinguished by relative errors made in one spectrum of the number line: the lower half (e.g., 0 to 50, 0 to 500). However, the poor estimators did not perform differently on the number spectrums for estimation lines 723 and 1,000. As can seen in table 18 there was not a consistent difference between spectrum results by the estimation groups as was found previously.

Per table 18, relative errors were significantly lower for the upper (M = 6.30, SE = .378) compared to lower (M = 4.76, SE = .309) spectrum for poor estimators in block A (100) ($F(1,357) = 9.928, p<.005$). In other words, poor estimators had significantly more errors in the lower end of the number spectrum than in the upper spectrum. In block B (1000), good estimators had significantly lower relative errors ($F(1, 619) = 4.411, p<.05$) on the lower spectrum (M = 3.37, SE = .169) than on the upper spectrum (M = 3.87, SE = .169). The relative error results for block C (723) show in table 18 that both good ($F(1, 516) = 8.957, p<.005$) and poor estimators ($F(1,298) = 5.123, p<.05$) were significantly different for each spectrum. Good estimators had lower relative errors in the lower spectrum (M = 3.98, SE = .188) compared to the upper spectrum (M = 4.86, SE = .225), but poor estimators had higher relative errors on the lower spectrum (M = 13.20, SE = .684) than the upper spectrum (M = 11.13, SE = .607).

A 3 (Estimator group: Good, Average and Poor) x 2 (Denomination: 100 and 723) x 2 (Spectrum: Lower and Upper) mixed design ANOVA was performed to further investigate the relationship between estimation ability, estimation line denomination and number spectrum on the estimation lines. Relative error was the dependent variable, estimator group was treated as the between subjects variable and denomination and spectrum were within-subjects variables. Significant main effects were found for estimator group ($F(1, 57) = 80.948$, $MSE = 4.219$, $p<.0005$, $\eta_p^2 = .740$) and denomination ($F(1, 57) = 108.347$, $MSE = 3.832$, $p<.0005$, $\eta_p^2 = .655$). Figures 34 and

35 both show the significant increase in relative errors as estimator group changes from good ($M = 4.330$, $SE = .201$) to average ($M = 5.544$, $SE = .236$) to poor ($M = 8.549$, $SE = .265$). Comparing the two figures, relative errors increased from the 100 denomination ($M = 4.792$, $SE = .178$) to the 723 denomination ($M = 7.490$, $SE = .197$). A two-way interaction between denomination and estimator group ($F(2, 57) = 18.013$, $MSE = 3.832$, $p < .0005$, $\eta_p^2 = .387$) was found. All three estimator groups had an increase in relative error from the 100 denomination (Good: $M = 3.786$, $SE = .264$, Average: $M = 4.479$, $SE = .309$, Poor: $M = 6.111$, $SE = .348$) to the 723 denomination (Good: 4.873, $SE = .292$, Average: $M = 6.608$, $SE = .341$, Poor: $M = 10.987$, $SE = .384$), but poor estimators' relative errors had a greater increase in the 723 denomination. A two-way interaction between spectrum and estimator group ($F(2, 57) = 4.709$, $MSE = 4.842$, $p < .05$, $\eta_p^2 = .142$) was also found. As figures 34 and 35 show, good and average estimators do not show much of a change in relative errors from lower ($M = 4.185$, $SE = .325$, $M = 5.534$, $SE = .380$, respectively) to upper ($M = 4.474$, $SE = .262$, $M = 5.554$, $SE = .306$, respectively) spectrum, but poor estimators had a decrease in relative errors from lower ($M = 9.459$, $SE = .428$) to upper ($M = 7.639$, $SE = .345$) spectrums. A two-way interaction between denomination and spectrum ($F(1, 57) = 22.886$, $MSE = 2.790$, $p < .01$, $\eta_p^2 = .12$) was also found. Relative errors were similar across lower ($M = 7.425$, $SE = .299$) and upper ($M = 7.554$, $SE = .308$) for 723 denominations, but decreased from the lower ($M = 5.360$, $SE = .240$) to the upper ($M = 4.223$, $SE = .183$) spectrums for the 100 denomination.

Estimation Groups' Reaction Time

The midpoint strategy is characterized by individuals accurately and rapidly estimating the midpoint of a number line, while at the same time experiencing difficulty (i.e., poor accuracy and longer reaction time) in estimating the quartile digits. By separating the estimators into poor, average and good estimator groups, we can show who is using this strategy. When reaction times are analyzed by estimator group, two findings become apparent: poor estimators are not fully using the midpoint strategy and good estimators are taking longer than either poor or average estimators to complete the task.

There were overall reaction time differences for all three estimator groups for each block as shown in table 19. Block A (100) reaction time results found an overall difference between the estimator groups ($F(1, 1439) = 19.855, p<.0005$) and that average estimators ($M = 2377, SE = 62$) were significantly faster than poor estimators ($M = 2890$, SE 106) and good estimators ($M = 3101, SE = 85$), $p<.0005$ for both. Block B (1000) reaction results again showed an overall estimator group difference in reaction times ($F(2, 1423) = 23.233, p<.0005$) and that again average estimators ($M = 2531, SE = 69$) were faster in responding than poor ($M = 3186, SE = 151$) or good ($M = 3455, SE = 92$) estimators, $p<.0005$ for both. Block C (723) reaction time results were also significantly different between the groups ($F(2, 1236) = 42.057, p<.0005$), but good estimators ($M = 4612, SE = 160$) separated themselves by responding much slower than both poor ($M = 3771, SE = 84$) or average ($M = 3000, SE = 79$) estimators, $p<.0005$ for both.

As table 21 shows, there were no significant differences between the two spectrums (i.e., lower and upper) within any of the three denominations for the three estimation groups.

A 3 (Estimator group: Good, Average and Poor) x 2 (Denomination: 100 and 723) x 2 (Spectrum: Lower and Upper) mixed design ANOVA was performed to further investigate the relationship between estimation ability, estimation line denomination and number spectrum on the estimation lines. Reaction time was the dependent variable, estimator group was treated as the between subjects variable and denomination and spectrum were within-subjects variables. A significant main effect was found for denomination ($F(1, 57) = 27.083$, $MSE = 1669433.498$, $p<.0005$, $\eta_p^2 = .322$), as seen by comparing figure 37 to 38, such that 100 denomination ($M = 2982.829$, $SE = 180.819$) had a much faster reaction time than the 723 denomination ($M = 3873.072$, $SE = 317.372$). An interaction between estimator group and denomination, as shown in figures 37 and 38, was found ($F(1, 57) = 4.179$, $MSE = 1669433.498$, $p<.05$, $\eta_p^2 = .128$). All three estimator groups increased in reaction time from the lower (Good: $M = 3318.235$, $SE = 267.843$, Average: $M = 2573.627$, $SE = 313.322$, Poor: $M = 3056.624$, $SE = 352.632$) to the upper (Good: $M = 4858.217$, $SE = 470.116$, Average: $M = 3225.483$, $SE = 549.939$, Poor: $M = 3535.514$, $SE = 618.936$) spectrums, however, good estimators' reaction time increased much more than average or poor estimators on the 723 denomination. An interaction between spectrum and denomination, as shown in both figures 37 and 38, was also found ($F(1, 57) = 7.563$, $MSE = 116537.457$, $p<.01$, $\eta_p^2 = .117$). Reaction time on 100 denomination decreased from the lower ($M = 3050.961$, $SE = 179.635$) to the upper ($M = 2914.696$, $SE = 186.522$) spectrum, but increased for the 723 denomination from lower ($M = 3816.909$, $SE = 298.208$) to upper ($M = 3929.234$, $SE = 342.285$) spectrum.

Estimation Groups' Directional Errors

Directional errors were also analyzed for these three groups of estimators on all three pure estimation line blocks.

For the 100 estimation lines, estimator groups had significantly different directional errors (F(2, 1541) = 12.995, MSE = 27.938, p<.0005) such that good estimators (M = .68, SE = .172) and average (M = .65, SE = .233) had lower directional errors than poor estimators (M = 2.24 SE = .336). In other words, poor estimators significantly overestimated the value of the mark compared to the good and average estimators. A similar result was found for block B (1000) with a significant difference in directional errors between the groups (F(2, 1522) = 4.503, MSE = 39.582, p<.05), specifically the good (M = -.39, SE = .175) and average (M = -.64, SE = .364) had much lower directional errors compared to poor estimators (M = .61, SE = .338). Finally, for block C (723) there was a significant difference on directional errors (F(2, 1311) = 163.245, MSE = 55.854, p<.0005)such that the good (M = 1.17, SE .222) and average (M = 3.81, SE = .373) estimators had lower directional errors(i.e., underestimates) than the poor estimators (M = 10.49, SE = .557). Table 22, bottom row, shows that good (M = 1.17), average (M = 3.81) and poor (M = 10.49) estimators all had positive directional errors (e.g., were overestimates), and table 23, bottom block, shows that all three groups were significantly different from each other, p<.0005 for each.

Table 24 shows a trend of overestimates of digits in the lower end of the spectrum and underestimates for digits in the upper end of the number line for blocks A (100) and B (1,000). However, you can now see that the poor estimators distinguish themselves by primarily having higher overestimates in the lower end and lower underestimates in the upper end of the number line. This trend is seen in Figure 39, panel A and B. Block C (723) is again unique: all of poor estimators' responses are gross overestimates, as see in figure 39, panel C. Another aspect of poor estimators' responses to 723 number lines is that they are equally poor for both ends of the spectrum and therefore not significantly different as shown in table 24.

A 3 (Estimator group: Good, Average and Poor) x 2 (Denomination: 100 and 723) x 2 (Spectrum: Lower and Upper) mixed design ANOVA was performed to further investigate the relationship between estimation ability, estimation line denomination and number spectrum on the estimation lines. Directional error was the dependent variable, estimator group was treated as the between subjects variable and denomination and spectrum were within-subjects variables. Significant main effects were found for estimator group ($F(1, 57) = 36.017$, $MSE = 12.138$, $p<.0005$, $\eta_p^2 = .558$), denomination ($F(1, 57) = 39.289$, $MSE = 11.260$, $p<.0005$, $\eta_p^2 = .408$) and spectrum ($F(1, 57) = 33.193$, $MSE = 16.500$, $p<.0005$, $\eta_p^2 = .368$). As estimator group changed from good ($M = 1.171$, $SE = 3.42$) to average ($M = 2.195$, $SE = .400$) to poor ($M = 5.893$, $SE = .450$) directional error increased as displayed in both figure 40 and 41. A Comparison of figures 40 to 41 shows that directional error increased as denomination increased from 100 ($M = 1.694$, $SE = .175$) to 723 ($M = 4.479$, $SE = 4.18$). Figure 40 shows directional error decreased from the lower ($M = 4.635$, $SE = .324$) to the upper ($M = 1.537$, $SE = .382$) spectrum. An interaction between estimator group and denomination, as shown in figures 40 and 41, was found ($F(1, 57) = 11.044$, $MSE = 11.260$, $p<.0005$, $\eta_p^2 = .279$), such that for all three estimator groups there was an increase in directional error from the 100 denomination (Good: $M = .842$, $SE = .259$, Average: $M = 1.212$, $SE = .303$, Poor:

$M = 3.028$, $SE = .341$) to the 723 denomination (Good: $M = 1.500$, $SE = .619$, Average: $M = 3.177$, $SE = .724$, Poor: $M = 8.759$, $SE = .814$), but poor estimators directional error means increase much more dramatically for the 723 denomination. A second interaction was found for denomination by spectrum ($F(1, 57) = 44.643$, $MSE = 7.400$, $p<.0005$, $\eta_p^2 = .439$). Both, 100 and 723 denominations saw a decrease in directional error as the spectrum changed from lower ($M = 4.446$, $SE = .311$ and $M = 4.825$, $SE = .580$, respectively) to upper ($M = -1.059$, $SE = .359$, $M = 4.133$, $SE = .517$, respectively) spectrums, but the 100 denomination saw a much greater decrease in directional error. A third and final interaction was found for estimator group by denomination by spectrum ($F(1, 57) = 3.150$, $MSE = 7.400$, $p \leq .05$, $\eta_p^2 = .100$). All three estimator groups and both denominations had a decrease in directional error from the lower to upper spectrums, except for poor estimators on the 723 denomination (Lower: $M = 8.630$, $SE = 1.131$, Upper: $M = 8.888$, $SE = 1.008$) as shown in figures 40 and 41.

Eye Data

Eye movements of subjects were recorded and analyzed during this experiment. As in pilot experiments, eye movements were divided into two data points: fixations and dwell time. Also, areas of the number line stimulus displayed to the subjects were subdivided into regions called Areas of Interest (AOIs) after data collection. An example of an estimation line with AOIs overlaid is in figure 42. There were 9 total AOIs created: start section, left middle section, middle section, right middle section, end section, upper section, lower section, the mark and an outer mark area. The first 7 sections were static and covered the entire monitor, but primarily focused on the length of the horizontal estimation line. The mark AOI moved along the estimation line with the actual mark as it changed values. The outer mark was an AOI added to ensure that the original and smaller mark AOI was accurately capturing fixations.

Total fixations were calculated for each of the first three, pure blocks of estimation lines (100, 1000 and 723). There was a significant difference in total fixations between the three blocks, $F(2, 3981) = 17.337$, $MSE = 210.231$, $p<.0005$), see table 25, final column. Subjects had longer reaction times from block A (100) to block B (1,000) to block C (723) so this finding was not unexpected. However, total fixations that subjects made were not very indicative of anything outside of an increased reaction time coincided with an increase in fixations.

A 3 (Math Anxiety Group: Low, Medium and High) x 2 (Denomination: 100 and 723) x 2 (Spectrum: Lower and Upper) mixed design ANOVA was performed to further investigate the relationship between math anxiety, estimation line denomination and number spectrum on the estimation lines. Total number of fixations was the dependent variable, math anxiety group was treated as the between subjects variable and denomination and spectrum were within-subjects variables. No significant main effects or interactions were found.

A 3 (Estimator group: Good, Average and Poor) x 2 (Denomination: 100 and 723) x 2 (Spectrum: Lower and Upper) mixed design ANOVA was performed to further investigate the relationship between estimation ability, estimation line denomination and number spectrum on the estimation lines. Total number of fixations was the dependent variable, estimator group was treated as the between subjects variable and denomination and spectrum were within-subjects variables. No significant main effects or interactions were found.

Table 25 shows a breakdown of where fixations were made along the number line, beginning with the start point of the line to the end point. Differences among the AOIs within the blocks are not readily apparent, but are found when number spectrum is used as a grouping variable.

When number spectrum (low versus high) is added as a dividing variable, differences in fixations are seen. As table 26 shows, there are far more fixations on the start and left middle sections when the mark is in the lower end of the spectrum. The very opposite holds true when the mark is in the upper end, there are far more fixations in the right middle and end sections. This may seem intuitive that individuals would look more at areas where the mark is located, but as you can see when fixations within AOIs are broken down by digit (appendix, table 2), individuals are primarily fixating on the start and end sections nearest to the mark.

Estimation Groups' Fixations

All three estimator groups made fewer fixations for all three blocks in the start AOI (Good: $M = .81$, $SE = .050$, Average: $M = .81$, $SE = .071$, Poor: $M = .71$, $SE = .050$) than any other AOI, with the end AOI having the most fixations (Good: $M = 2.56$, $SE = .097$, Average: $M = 2.05$, $SE = .76$, Poor: $M = 1.97$, $SE = .079$), as shown in table 27. A one-way ANOVA was performed for the number of fixations within each AOI by estimator group. Good estimators ($M = 2.64$, $SE = .079$) made more fixations than average ($M = 2.13$, $SE = .086$) or poor ($M = 2.03$, $SE = .099$) estimators in the middle AOI ($F(2, 3980) = 15.280$, $MSE = 10.092$, $p < .0005$). Good estimators ($M = 2.26$, $SE = .100$) made more fixations than poor ($M = 1.59$, $SE = .098$) or average ($M = 1.48$, $SE = .071$) estimators in the right middle AOI ($F(2, 3980) = 21.542$, $MSE = 12.189$, $p < .0005$.

Good estimators ($M = 2.56$, $SE = .097$) made more fixations than average ($M = 2.05$, $SE = .076$) or poor ($M = 1.59$, $SE = .098$) estimators in the end AOI ($F(2, 3980) = 13.484$, $MSE = 10.972$, $p < .0005$). These differences coincide with the previous reaction time differences between the estimator groups that found that good estimators took longer to respond.

Block A (100)

All three estimator groups made fewer fixations for block A (100) in the start AOI (Good: $M = .94$, $SE = .087$, Average: $M = .84$, $SE = .131$, Poor: $M = .89$, $SE = .106$) than in any other AOI, with the end AOI having the most fixations (Good: $M = 2.34$, $SE = .103$, Average: $M = 2.01$, $SE = .134$, Poor: $M = 2.13$, $SE = .141$), as shown in table 28. The middle AOI also has a high number of fixations (Good: $M = 2.22$, $SE = .095$, Average: $M = 2.20$, $SE = .164$, Poor: $M = 2.14$, $SE = .208$), but this may be due to subjects beginning each trial with a fixation in this AOI. A one-way ANOVA was performed for the total number of fixations, with no significant difference between the estimator groups. This was not unexpected due to the minor differences between the estimator groups in reaction time.

The consistent directional error trend of over- then underestimates in the 100 number line results discussed earlier leads to a hypothesis involving fixations and their locations. It was thought that the over- and underestimates may be due to comparing the mark with either the start AOI of the estimation line (underestimates) or end AOI of the estimation line (underestimates). This hypothesis is supported with a significant negative correlation between the number of fixations in the start AOI or end AOI and mean directional error. Good and poor estimators both had significant correlations between start AOI fixations and directional error (r = .147, $p<.01$ and r = .140, $p<.01$, respectively), meaning that as fixations in the start AOI increased, directional errors went up or were overestimates. The good and poor estimators also had significant correlations between the end AOI fixations and directional error (r = -.175, $p<.01$ and r = -.130, $p<.05$, respectively), meaning that as fixations in the end AOI increased, directional error went down or were underestimates.

Block C (723)

Subjects made fewer fixations on the start AOI than in any other AOI for block C (723), as shown in table 29. Unlike in block A (100) fixation results, estimator groups were significantly different in the total number of fixations made in certain AOI's. Left middle ($F(2, 1433) = 4.096$, $MSE = 7.325$, $p<.05$), middle ($F(2, 1433) = 11.223$, $MSE = 9.290$, $p<.0005$) and right middle ($F(2, 1433) = 10.552$, $MSE = 13.667$, $p<.0005$) AOI's all found that good estimators made more fixations in them than poor estimators. A greater number of fixations by the good estimators compared to the poor estimators were expected, due to the longer reactions times for the good compared to the average and poor estimators. Poor estimators bore out the directional error and fixation location hypothesis with a significant correlation (r = -.122, $p<.05$) between directional errors and number of end AOI fixations. In other words, the fewer fixations made at the end AOI, the more positive (e.g., overestimates) the directional error mean was, and the poor estimators made far fewer fixations in the end AOI and had much higher positive directional errors.

Good estimators ($M = 1.70$, $SE = .170$) made more fixations than poor estimators ($M = .85$, $SE = .105$) in the left half of the estimation line when the mark was in the upper spectrum ($F(2, 556) = 9.671$, $MSE = 4.599$, $p<.0005$). With the previously established differences in relative and directional errors, and reaction times between the good and poor estimators, we can infer that the good estimators had lower relative errors and fewer underestimates due to the increase in fixations at the opposite end of the number line, which may have provided a better understanding of the total length of the line and the location of the mark. Good estimators ($M = 3.51$, $SE = .315$) also made more fixations than poor estimators ($M = 1.86$, $SE = .153$) in the right half of the estimation line when the mark was in the lower spectrum ($F(2, 763) = 13.492$, $MSE = 18.519$, $p<.0005$). This difference in fixations between the estimator groups points to good estimators using the entire line to determine their response; if the mark was on the left side of the estimation line then good estimators fixated more frequently on the right side.

Final Fixation

The last fixations subjects made per trial were analyzed to show at what and where they were looking when their response was made. This would provide a window into which object or aspect of the number line subjects used as their final piece of data prior to their answer. We removed trials that were outliers and trials in which individuals made fewer than 4 fixations. Over half of all trials (Block A: 976 (62.6%), Block B: 968 (62.1%), and Block C: 800 (51.3%) had at least four fixations and it is hypothesized that four fixations would be the least number of fixations necessary in order to characterize a cognitive strategy. As can be seen in the results for all three blocks in table 30, almost half of all final fixations were within the mark AOI.

Prior to describing the actual fixation sequencing results, a group of predictions is needed. Due to the directional error patterns found, we predicted a specific group of fixations. Overestimates of digits in the lower end of the spectrum may be due to a comparison made between the beginning of the line and the actual mark on the estimation line. This comparison would involve a fixation made at the start AOI and then a second fixation at the mark AOI. Conversely, underestimates found for digits in the upper spectrum would show a fixation at the end of the number line followed by a fixation at the mark. Both of these fixation sequences involve a comparison between the mark and the closest end point (e.g., start of the line or the end of the estimation line), and more specifically an overestimation of the distance between the closest end point and the mark. In other words, in the lower number spectrum, the directional errors are obviously overestimates. However, in the upper spectrum, despite the directional errors being underestimates, they are still overestimates, in that they are further in distance from the end of the estimation line. A caveat is needed when dealing with fixations on the end AOI: subjects made at least one fixation in the end AOI to determine the numeric magnitude of the estimation line, thus some of the end fixations would not be for the operation of comparison between the end AOI and mark.

By looking at the final and penultimate fixation in Block A's fixation sequencing results we can determine the final comparison between areas on the number line used by individuals when completing this task. More than half of all trials (490, 50.2%) ended with a fixation on the mark AOI, but the next to last fixation varied depending on the location of the mark on the number line. As we have seen, changes in the directional errors depending on whether the mark was in the lower end of the number spectrum or in the upper end of the number spectrum, we have also found differences in the penultimate fixation. Block A's fixation sequencing results are a good baseline to compare to Block

C, due to its homogenous results across estimation groups. A picture of individuals' final determination of the value of the mark becomes clear as the fixation results are narrowed down to the trials in which the mark AOI is the final fixation and the actual mark on the number line is in the upper end of the number spectrum. The most frequent next to last fixation is the end AOI with 55.9 percent of the fixations, while the start AOI is the least fixated AOI with 1.4 percent. This pattern is not replicated when the mark on the number line is in the lower spectrum; the second to last fixation is the start AOI 22.9% of the time and only 21.4% of the time in the end AOI. These two results, when the mark is in the lower versus upper spectrum, coincide with the directional error results for Block A from earlier. These findings are the basis for a theory about the cognitive process that adults use to complete this estimation task. Adults tend to compare the end of the number line closer to the mark to make their final judgment, which causes them to either overestimate, when the mark is in the lower spectrum, or underestimate, when the mark is in the upper spectrum.

Block C's sequencing results are very similar to Block A's, which points to a similar strategy used no matter the denomination. Only 293 (36.6%) of the trials in block C ended with the mark AOI as the final fixation. Examining only the trials in which the final fixation was made in the mark AOI and dividing the results between the two number spectrums we found the same pattern of fixation sequence as in block A. When the mark was in the lower spectrum, individuals' penultimate fixation was made 18.5 % of the time in the start AOI and 29.6% of the time in the end AOI. When the mark was in the upper end of the number spectrum, the penultimate fixation was made only 1.6 percent of the time in the start AOI, but 61.2 percent of the time in the end AOI.

When we divide the fixation sequencing results by estimator group in block C, we expected to find decided differences due to previous directional error results, with the poor estimators having much greater overestimates for all digits compared to good or average estimators. It was expected that poor estimators would have many more fixations in the end AOI than the good estimators for digits in the lower and upper spectrum, which would coincide with the overestimates of all digits. This did not happen. Both poor and good estimators' eye fixations continued in the pattern from block A, with fewer fixations in the start (poor: 11.4%, good: 10.5%) AOI than the end (poor: 22.9%, good: 33.3%) AOI in the lower spectrum digits and even fewer fixations in the start (poor: 2.9%, good: 0%) AOI than the end (poor: 70.6%, good: 57.9%) AOI in the upper spectrum.

Summary of Results

The current experiment replicated pilot experiment results using similar stimuli: 0 to 100, 0 to 1000 and 0 to 723 estimation lines. Adults responded in an overwhelmingly linear fashion when completing all three denominations of number lines, but their responses had many distinct and consistent characteristics. Errors and reaction times allowed us to examine how well and quickly subjects estimated the value of the mark. Relative errors for all three denominations exhibited an "M" shape when graphed: high accuracy at the midpoint and endpoints of the number line and poorer accuracy at the quartiles. Reaction times for 100 and 1000 denominations paralleled relative error patterns with quick reaction times at the midpoint and endpoints, and slower reaction times at the quartiles. However, the longer reaction times at the quartiles do not appear to increase accuracy. These two measures' results, relative error and reaction time, could be interpreted to mean that individuals need very little time to accurately determine the midpoint of a line with a familiar magnitude (e.g., halve it), but struggle to find the quartile (e.g., halve it again). Relative error and reaction time trends were not exactly the same for the 723 denomination: there was no characteristic dip in reaction time at the midpoint. This "increase" in reaction time on the 723 estimation lines, compared to the 100 and 1000 lines, is thought to be due to the additional operation of dividing 723 in half. Subjects may quickly and accurately make the judgment that the mark is at the midpoint of the number line, but then have to perform the halving of the line's magnitude.

Directional errors gave a third characteristic about subjects' responses: whether they were over- or underestimating. Once again 100 and 1000 estimation lines shared response characteristics, but in this case it was a trend of overestimates in the lower half of the number line (e.g., digits below the midpoint) and underestimates in the upper half of the number line (e.g., digits above the midpoint). Nearly every subject's response to the mark when it was below the midpoint was an overestimate and when the mark was above the midpoint it was an underestimate. The 723 denomination results were again quite different from the 100 and 1000 denomination results. Nearly every subject overestimated the mark on the number line for all digits, not just the lower spectrum of numbers.

In order to investigate the effects of math anxiety, we analyzed the three math anxiety groups' relative error, reaction time and directional error results. Interestingly, high math anxious individuals performed differently than low or medium math anxious individuals on this very elementary task. All three math anxiety groups performed similarly on 100 and 1000 denomination relative errors, with an "M" shape for all three groups. However, low math anxious subjects had noticeably longer reaction times than medium or high math individuals, following previous research that shows high math anxious subjects hurry through math tasks for avoidance purposes. This is surprising due to the lack of extensive, mathematical processes needed to complete this task. The directional errors for the 100 and 1000 number lines were consistently over- then underestimates for all three math anxiety groups. The 723 denomination results were once again quite different from the two previous ones. High math anxious individuals distinguished themselves from medium and low anxious individuals by not having an increase in accuracy at the midpoint. The high math anxious group was not able to accurately estimate the midpoint of the 723 number lines, unlike the low and medium math anxiety groups. The high math anxious group completed these number lines much more quickly than the medium and low math anxious groups as well, further supporting the avoidance theory of math anxiety. Another difference between the math anxiety groups was found in the directional error results, with high math anxious subjects making much higher overestimates for all digits in the 723 denomination in comparison either the medium or low math anxious individuals.

Estimator groups bore similar results, if not with starker differences. Again, all three estimator groups had similar relative error results for 100 and 1000 number lines and showed the familiar "M" shaped graph. Reaction times also showed an "M" shape and highlighted the fact that individuals were able to very quickly estimate the midpoint of the number line; however good estimators like low math anxious individuals had slightly longer reaction times than the other groups for 100 and 1000 denominations. The

723 denomination divided the three estimator groups and showed the poor estimators' inability to find the midpoint of this denominations line. Reaction time results for this block also showed that poor and average estimators took less time to make estimations in this denomination.

Finally, the eye movement data showed that individuals used a uniform strategy when completing this estimation line task. Subjects consistently ended by making a fixation on the mark and used the end of the estimation line closer to the mark to make their final comparison. This final comparison may be an explanation for the repeated over- and underestimates found in the directional errors: subjects overestimate the value of the mark when comparing it to the start of the number line and underestimate the value of the mark when comparing it to the end of the number line.

Discussion

The purpose of this thesis was to gain insight into the cognitive processes used when people make numerical estimates in the line estimation task. As noted in the introduction, this is a simple, unit-free task that has been used widely with school-aged children. The pattern of children's responses has been shown to reveal whether children's mental representation of number is basically a logarithmic number line, with unequally spaced intervals along the number line, or whether the mental representation has matured into a linear representation, with equally spaced intervals. As explained earlier, while this work is important, it still does not address the issue of underlying mental processes involved in making numerical estimates. That is, even if a child (or adult) has reached an understanding of number that is characterized by a linear representation of numerical quantity, this still does not tell us how that representation is applied when actually estimating numerical values in any particular setting.

Given the results obtained in this research, we can now address some of these questions about cognitive processes in numerical estimation.

How did people make their estimates?

First, adults' responses have shown that they have a linear mental number line. Second, there are areas of the number line that they can estimate very accurately and efficiently and other areas that they cannot. Having overall linear responses does mean relative accuracy, but an in depth look, beyond the overall relationship of the responses to each other, finds that there are distinct differences in errors throughout the number line. Adults are able estimate the midpoint very quickly and very accurately, leading us to believe that bisecting a line is a widespread strategy that adults with a complete number sense employ during estimation. However, a second order bisection is not performed, as shown by the inability of adults to replicate the efficiency at which they estimate the midpoint to the quartiles. In sum, it appears that adults use three reference points to make estimations: the start and end of the number line, and the actual mark itself. Other than these three areas, adults display difficulty in making accurate estimations.

Is there any evidence of a Dehaene-like (and Siegler found) logarithmic number line mental representation?

The adults in our sample were overwhelmingly linear in their responses, but their errors had as surprising a quality as children displayed: nearly all of the adults' errors were overestimates of digits in the lower half of a number line and underestimates of digits in the upper half of a number line.

Was there a single strategy or multiple estimation strategies used?

Along with the consistent relative and directional errors adults made, their eye movements had a consistent pattern as well. The strategy used does not appear to be a methodical, laying out of a ruler on the number line and counting off digits (from left to right) until reaching the mark. A more difficult partitioning (e.g., halving or creating thirds) of the number line does not appear in the eye movements either. Adults used the start and end areas on the number line and the mark itself as reference points to complete this estimation task. They specifically appear to use the end of the number line closer to the mark to make their final determination of the value of the mark. They seem to estimate, or "measure subjectively", from the closest endpoint, where those estimates tend to be over- not underestimates. However, this subjective strategy is abandoned for a more mathematical process if the mark is very close to the well-known midpoint.

Did math anxiety have an effect on the performance of this simple estimation task?

Math anxiety appears to have detrimental effects for individuals completing even this elementary task. High math anxious individual complete this task much more quickly than low or medium individuals, with commensurately high errors. This culminates in the 723 number line relative error results, in which high math anxious individuals have difficulty even estimating the midpoint. It is possible that the high math anxious individuals are driven to complete this task in such a hurry, due to their dislike of even a rudimentary math task, that they don't care whether their estimation is accurate. It is also possible that their years of math avoidance and online math anxious thoughts are increasing the difficulty of computing the division of the number line

APPENDIX

TABLES AND FIGURES

Table 1. Demographic, sMARS and WRAT results

	Math Anxiety Groups (mean/SD)			
Demographic Variable	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**	
				Sig
Gender (M/F)	7/13	5/9	3/8	
Age	19.95/3.79	21.07/6.27	20.55/6.89	NS
Class Year	1.95/1.15	1.86/1.10	1.27/.65	NS
Number of H.S. math courses taken	3.45/.76	3.62/.77	3.18/1.08	NS
H.S. math grade	3.15/.813	3.29/.73	2.45/.69	0.05
Number of college math courses taken	1.55/1.73	1.50/1.79	.91/1.22	NS
College math grade	3.08/.64	3.40/.97	2.17/1.17	0.05
Rated math anxiety	3.30/2.47	4.36/1.91	5.45/2.12	0.05
Rated math enjoyment	5.30/2.60	6.00/2.54	4.45/2.38	NS
sMARS score	12.55/5.75	34.79/5.01	58.45/5.24	0.0005
WRAT score	28.11/5.64	31.93/4.62	27.45/3.96	0.05
Ethnic Group % of total				NS
African-American	10	7.1	18.2	
Hispanic/Latino	15			
Native American	5			
Asian/Pacific Islander	10	35.7	27.3	
Caucasian	50	42.9	36.4	
Other	5	7.1	18.2	

Table 2 Microphone errors and Outliers

	Microphone Errors	%	Outliers	%	Incorrect Denomination	%
Block A	113	7.24	26	1.67	0	0.00
Block B	100	6.41	20	1.28	0	0.00
Block C	113	9.42	8	0.51	0	0.00
Block D	97	6.22	53	3.40	226	14.49
Block E	84	5.83	12	0.77	0	0.00
Block F	113	7.85	21	1.35	17	1.09
Total	620	6.62	140	1.50	243	2.60

Table 3. Table of Responses' Linearity

	Linear		Logarithmic
	r^2	N >.90	r^2
100	0.971	1	0.888
1000	0.966	2	0.898
723	0.937	10	0.853

Table 4. Relative, Directional and Reaction Times All Blocks

Blocks	Relative Errors M (SE)	Directional Errors M (SE)	Reaction Times M (SE)
A (100)	4.14 (.090)	1.06 (.136)	2826 (49)
B (1000)	4.49 (.114)	-.22 (.162)	3100 (58)
C (723)	7.09 (.171)	4.34 (.230)	3793 (82)
Block D (100/1000)	4.33 (.109)	.38 (.161)	3226 (59)
Block E (723/1000)	5.34 (.135)	1.41 (.192)	3694 (71)
Block F (100/723)	5.60 (.158)	2.48 (.209)	3598 (76)

Table 5. Relative, Directional Errors and Reaction Times for Pure vs. Mixed blocks

Blocks	Relative Errors M (SE)	Directional Errors M (SE)	Reaction Times M (SE)
100 (Block A)	4.14 (.090)	1.06 (.136)	2825 (49)
100 (Block D)	4.42 (.145)	1.40 (.213)	3084 (73)
100 (Block F)	4.84 (.199)	2.17 (.254)	3129 (77)
1000 (Block B)	4.49 (.114)	-.23 (.162)	3100 (58)
1000 (Block D)	4.22 (.166)	-.82 (.236)	3394 (95)
1000 (Block E)	4.22 (.152)	-.41 (.214)	3297 (76)
723 (Block C)	7.09 (.171)	4.34 (.230)	3792 (82)
723 (Block E)	6.68 (.223)	3.58 (.314)	4170 (125)
723 (Block F)	6.56 (.249)	2.86 (.348)	4209 (140)

Table 6. Relative, Directional Errors and Reaction Times per block and spectrum

Blocks	Spectrum	Relative Errors M (SE)	F	Directional Errors M (SE)	F	Reaction Times M (SE)	F
A (100)	Lower	4.57 (.143)	(1,1421) = 7.113	3.60 (.178)	(1,1421) = 361.293	2869 (72)	(1, 1398) = .131
	Upper	4.06 (.125)	*p*<.001	-1.36 (.191)	*p*<.0005	2833 (70)	NS
B (1000)	Lower	4.55 (.158)	(1,1404) = 1.115	2.27 (.218)	(1,1404) = 245.593	3175 (84)	(1,1384) = .022
	Upper	4.80 (.170)	NS	-2.66 (.227)	*p*<.0005	3157 (88)	NS
C (723)	Lower	7.22 (.267)	(1, 1193) = .032	5.02 (.341)	(1,1192) = 6.527	3765 (116)	(1,1182) = .218
	Upper	7.16 (.238)	NS	3.78 (.344)	*p*<.05	3846 (129)	NS
D							
100	Lower	5.05 (.249)	(1, 665) = 8.506	4.25 (.291)	(1, 665) = 180.547	3283 (111)	(1, 630) = 2.811
	Upper	4.15 (.181)	*p*<.005	-1.20 (.283)	*p*<.0005	3025 (106)	NS
1000	Lower	4.11 (.262)	(1, 565) = 2.713	.60 (.361)	(1,565) = 32.167	3459 (135)	(1, 530) = .018
	Upper	4.69 (.236)	NS	-2.20 (.337)	*p*<.0005	3432 (145)	NS
E							
723	Lower	6.41 (.333)	(1, 592) = 2.318	3.98 (.443)	(1, 592) = 1.010	4177 (187)	(1, 561) = .043
	Upper	7.11 (.318)	NS	3.32 (.486)	NS	4232 (189)	NS
1000	Lower	4.31 (.240)	(1, 716) = .793	1.10 (.326)	(1, 716) = 49.10	3499 (116)	(1, 678) = 2.120
	Upper	4.60 (.215)	NS	-2.03 (.306)	*p*<.0005	3263 (113)	NS
F							
100	Lower	6.02 (.381)	(1, 691) = 20.171	5.19 (.415)	(1, 691) = 132.357	3295 (105)	(1, 653) = 1.487
	Upper	4.14 (.182)	*p*<.0005	-.57 (.285)	*p*<.0005	3095 (124)	NS
723	Lower	6.27 (.363)	(1, 533) = 2.229	2.80 (.515)	(1, 533) = .043	4138 (195)	(1, 489) = .466
	Upper	7.05 (.372)	NS	2.96 (.526)	NS	4346 (220)	NS

Table 7. Math Anxiety Groups' Relative Errors

		Anxiety Group				
Block	M (SE)	Low (N=20)	Medium (N=14)	High (N =11)	F	*p*<
A (100)	4.14 (.090)	4.07 (.181)	3.71 (.143)	4.63 (.207)	(2, 1154) = 5.177	.001
B (1000)	4.49 (.114)	4.20 (.151)	3.76 (.165)	4.19 (.208)	(2, 1133) = 2.154	NS
C (723)	7.09 (.171)	6.70 (.293)	5.27 (.249)	9.29 (.472)	(2, 981) = 30.928	.0005
D (overall)	4.33 (.109)	4.12 (.182)	3.87 (.186)	4.73 (.254)	(2, 1004) = 3.804	.05
D (100)	4.42 (.145)	4.23 (.241)	3.97 (.259)	4.93 (.316)	(2, 546) = 2.812	NS
D (1000)	4.22 (.166)	3.99 (.277)	3.75 (.268)	4.47 (.419)	(2, 457) = 1.057	NS
E (overall)	5.34 (.135)	5.25 (.220)	4.27 (.213)	6.67 (.399)	(2,1070) = 16.906	.0005
E (723)	6.68 (.223)	6.18 (.333)	4.98 (.353)	9.09 (.655)	(2, 485) = 20.133	.0005
E (1000)	4.22 (.166)	4.47 (.284)	3.68 (.251)	4.63 (.415)	(2, 584) = 2.408	NS
F (overall)	5.60 (.158)	4.98 (.247)	4.78 (.375)	6.56 (.365)	(2,997) = 7.757	.0005
F (100)	4.84 (.199)	4.47 (.351)	4.51 (.486)	5.31 (.382)	(2, 554) = 1.163	NS
F (723)	6.56 (.249)	5.62 (.339)	5.13 (.585)	8.15 (.646)	(2, 442) = 8.901	.0005

Table 8. Math Anxiety Groups' Relative Error ANOVA on Pure Blocks

Math Anxiety Groups' Relative Error ANOVA			
Block A (100)	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low			
Medium			$p<0.05$
High		$p<0.05$	
Block B (1000)			
Low			
Medium			
High			
Block C (723)			
Low		$p<0.05$	$p<0.0005$
Medium	$p<0.05$		$p<0.0005$
High	$p<0.0005$	$p<0.0005$	
Block D (overall)			
Low			
Medium			
High			
Block E (overall)			
Low			
Medium			$p<0.001$
High		$p<0.001$	
Block F (overall)			
Low			
Medium			$p<0.05$
High		$p<0.05$	

Table 9. Math Anxiety Groups' Relative Error ANOVA on Mixed Blocks

Math Anxiety Groups' Relative Error ANOVA			
Block D (100)	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low			
Medium			
High			
Block D (1000)	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low			
Medium			
High			
Block E (723)	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low			0.01
Medium			0.0005
High	0.01	0.0005	
Block E (1000)	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low			
Medium			
High			
Block F (100)	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low			
Medium			
High			
Block F (723)	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low			0.01
Medium			0.005
High		0.005	

Table 10. Reaction Time by Math Anxiety Group per Block

Block	M (SE)	Low(N=20)	Medium(N=14)	High(N =11)	F	*p*<
			Anxiety Group			
A (100)	2825 (48)	3178 (92)	2925 (102)	2644 (93)	(2,1139) = 7.096	.001
B (1000)	3100 (57)	3607 (117)	3175 (113)	3007 (133)	(2, 1117) = 6.521	.005
C (723)	3792 (81)	4783 (189)	3801 (161)	3218 (105)	(2, 971) = 20.880	.0005
D (overall)	3226 (59)	3825 (122)	3003 (105)	2920 (86)	(2, 945) = 20.191	.0005
D (100)	3084 (73)	3619 (152)	2806 (131)	2971 (121)	(2, 516) = 9.810	.0005
D (1000)	3394 (94)	4060 (195)	3239 (169)	2850 (110)	(2, 428) = 11.142	.0005
E (overall)	3693 (71)	4296 (146)	3947 (166)	3383 (113)	(2, 1012) = 8.713	.0005
E (723)	4170 (125)	5149 (270)	4459 (280)	3542 (154)	(2,457) = 9.021	.0005
E (1000)	3297 (76)	3608 (134)	3519 (192)	3248 (163)	(2, 554) = 1.198	NS
F (overall)	3597 (76)	4109 (136)	3803 (207)	3459 (153)	(2, 924) = 3.718	.05
F (100)	3129 (77)	3529 (142)	3232 (199)	3309 (96)	(2, 520) = 2.381	NS
F (723)	4208 (139)	4896 (245)	4519 (387)	3995 (276)	(2, 403) = 2.112	NS

Table 11. Reaction Time by Math Anxiety Group per Pure Block ANOVA

Math Anxiety Groups' Reaction Times ANOVA			
Block A	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low			$p<.0.0005$
Medium			
High	$p<0.0005$		
Block B	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low		$p<0.05$	$p<0.0005$
Medium	$p<0.05$		
High	$p<0.0005$		
Block C	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low		$p<0.0005$	$p<0.0005$
Medium	$p<0.0005$		$p<0.005$
High	$p<0.0005$	$p<0.005$	
Block D	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low		$p<0.0005$	$p<0.0005$
Medium	$p<0.0005$		
High	$p<0.0005$		
Block E	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low			$p<0.0005$
Medium			$p<0.05$
High	$p<0.0005$	$p<0.05$	
Block F	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low			$p<0.05$
Medium			
High	$p<0.05$		

Table 12. Reaction Time by Math Anxiety Group per Mixed Block ANOVA

Math Anxiety Groups' Relative Error ANOVA			
Block D 100	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low		$p<0.005$	$p<0.05$
Medium	$p<0.005$		
High	$p<0.05$		
Block D 1000	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low		$p<0.001$	$p<0.0005$
Medium	$p<0.001$		
High	$p<0.0005$		
Block E 723	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low			$p<0.0005$
Medium			$p<0.0005$
High	$p<0.0005$	$p<0.0005$	
Block E 1000	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low			
Medium			
High			
Block F 100	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low			
Medium			
High			
Block F 723	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low			
Medium			
High			

Table 13. Math Anxiety Groups' Directional Errors per Block

Block	M (SE)	Anxiety Group: Low (N=20)	Medium (N=14)	High (N=11)	F	*p*<
A (100)	1.06 (1.36)	.69 (.252)	.63 (.241)	2.38 (.315)	(2,1154) = 11.155	.0005
B (1000)	-.23 (.162)	.12 (.239)	-.11 (.258)	.68 (.331)	(2, 1131) = 1.743	NS
C (723)	4.34 (.230)	3.34 (.405)	2.17 (.371)	7.60 (.584)	(2, 983) = 33.555	.0005
D (overall)	.38 (.161)	.09 (.266)	-.09 (.288)	1.61 (.383)	(2, 1004) = 7.618	.001
D (100)	1.40 (.213)	.99 (.359)	.92 (.396)	1.96 (.498)	(2, 546) = 1.725	NS
D (1000)	-.82 (.236)	-.94 (.385)	-1.27 (.399)	1.12 (.599)	(2,457) = 6.581	.005
E (overall)	1.41 (.192)	.58 (3.25)	.49 (.316)	3.88 (8.458)	(2, 1070) = 22.059	.0005
E (723)	3.58 (.314)	2.39 (.512)	1.89 (.517)	7.33 (.818)	(2, 485) = 21.132	.0005
E (1000)	-.41 (.214)	-.92 (.393)	-.65 (.369)	.96 (.563)	(2, 584) = 4.702	.01
F (overall)	2.48 (.209)	1.79 (.333)	1.39 (.457)	4.52 (.473)	(2, 997) = 14.492	.0005
F (100)	2.17 (.254)	1.59 (.442)	1.73 (.582)	3.23 (.522)	(2, 554) = 2.734	NS
F (723)	2.86 (.348)	2.05 (.505)	.96 (.727)	6.18 (.822)	(2, 442) = 14.528	.0005

Table 14. Math Anxiety Groups' Directional Errors per Overall Block ANOVA

Math Anxiety Groups' Directional Error ANOVA			
Block A	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low			$p<.0005$
Medium			$p<.0005$
High	$p<.0005$	$p<.0005$	
Block B	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low			
Medium			
High			
Block C	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low			0.0005
Medium			0.0005
High	0.0005	0.0005	
Block D	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low			$p<.005$
Medium			p=.001
High	$p<.005$	p=.001	
Block E	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low			$p<.0005$
Medium			$p<.0005$
High	$p<.0005$	$p<.0005$	
Block F	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low			$p<.0005$
Medium			$p<.0005$
High	$p<.0005$	$p<.0005$	

Table 15. Math Anxiety Groups' Directional Errors per Mixed Block ANOVA

Math Anxiety Groups' Relative Error ANOVA			
Block D 100	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low			
Medium			
High			
Block D 1000	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low			p=.005
Medium			p<.005
High	p=.005	p<.005	
Block E 723	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low			p<.0005
Medium			p<.0005
High	p<.0005	p<.0005	
Block E 1000	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low			p<.01
Medium			
High	p<.01		
Block F 100	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low			
Medium			
High			
Block F 723	**Low (n = 20)**	**Medium (n = 14)**	**High (n = 11)**
Low			p<.0005
Medium			p<.0005
High	p<.0005	p<.0005	

Table 16. Estimator Groups' Relative Errors by Block

		Estimator Group				
Block	M (SE)	Good (N=26)	Average (N=19)	Poor (N=15)	F	*p*<
A (100)	4.29 (.162)	3.58 (.145)	4.02 (.216)	5.25 (.340)	(2, 77) = 12.207	.0005
B (1000)	4.66 (.191)	3.48 (.197)	5.51 (.317)	4.99 (.328)	(2, 77) = 13.524	.0005
C (723)	7.81 (.484)	4.26 (.220)	6.89 (.239)	12.26 (.701)	(2, 65) = 83.433	.0005

Table 17. Estimator Groups' Relative Errors ANOVA

Estimator Groups' Relative Error ANOVA			
Block A	**Good (N=26)**	**Average (N=19)**	**Poor (N=15)**
Good			p<.0005
Average			p<.005
Poor	p<.0005	p<.005	
Block B	**Good (N=26)**	**Average (N=19)**	**Poor (N=15)**
Good		p<.0005	p<.001
Average	p<.0005		
Poor	p<.001		
Block C	**Good (N=26)**	**Average (N=19)**	**Poor (N=15)**
Good		p<.0005	p<.0005
Average	p<.0005		p<.0005
Poor	p<.0005	p<.0005	

Table 18. Estimator Groups' Relative Errors by Denomination by Spectrum

Estimator Groups' Relative Error ANOVA						
	Good	**F**	**Average**	**F**	**Poor**	**F**
Block A	M (SE)		M (SE)		M (SE)	
Lower	3.87 (.167)	NS	4.15 (.217)	NS	6.30 (.378)	(1, 357) = 9.928
Upper	3.52 (.149)		4.25 (.228)		4.76 (.309)	*p*<.005
Block B						
Lower	3.37 (.169)	(1, 619) = 4.411	5.42 (.351)	NS	5.58 (.307)	NS
Upper	3.87 (.169)	*p*<.05	5.93 (.400)		5.01 (.326)	
Block C						
Lower	3.98 (.188)	(1, 516) = 8.957	6.95 (.355)	NS	13.20 (.684)	(1, 298) = 5.123
Upper	4.86 (.225)	*p*<.005	7.15 (.367)		11.13 (.607)	*p*<.05

Table 19. Reaction Time by Estimator Groups

		Estimator Group				
Block	M (SE)	Good (N=26)	Average (N=19)	Poor (N=15)	F	*p*<
A (100)	2822 (50)	3101 (85)	2377 (62)	2890 (106)	(2, 1439) = 19.855	.0005
B (1000)	3099 (60)	3455 (92)	2531 (69)	3186 (151)	(2, 1423) = 23.233	.0005
C (723)	3771 (84)	4612 (160)	3000 (79)	3771 (84)	(2, 1236) = 42.057	.0005

Table 20. Reaction Time by Estimator Groups ANOVA

Estimator Groups' Relative Error ANOVA			
Block A	**Good (N=26)**	**Average (N=19)**	**Poor (N=15)**
Good		$p<.0005$	
Average	$p<.0005$		$p<.0005$
Poor		$p<.0005$	
Block B	**Good (N=26)**	**Average (N=19)**	**Poor (N=15)**
Good		$p<.0005$	
Average	$p<.0005$		$p<.0005$
Poor		$p<.0005$	
Block C	**Good (N=26)**	**Average (N=19)**	**Poor (N=15)**
Good		$p<.0005$	$p<.0005$
Average	$p<.0005$		
Poor	$p<.0005$		

Table 21. Estimator Groups' Reaction Time by Denomination by Spectrum

Estimator Groups' Reaction Times ANOVA						
	Good	F	Average	F	Poor	F
Block A	**M (SE)**		**M (SE)**		**M (SE)**	
Lower	3050 (118)	(1, 585) = 1.091	2469 (102)	(1, 416) = 1.017	3101 (172)	(1, 327) = 2.780
Upper	3233 (129)	NS	2334 (80)	NS	2735 (135)	NS
Block B						
Lower	3517 (131)	(1, 584) = .068	2554 (106)	(1, 413) = .009	3340 (233)	(1, 309) = .216
Upper	3568 (145)	NS	2567 (96)	NS	3192 (215)	NS
Block C						
Lower	4654 (227)	(1, 488) = .114	2983 (120)	(1, 355) = .276	3062 (171)	(1, 279) = 1.328
Upper	4770 (254)	NS	2900 (102)	NS	3384 (140)	NS

Table 22. Directional Errors by Estimator Groups

Block	M (SE)	Good (N=26)	Estimator Group Average (N=19)	Poor (N=15)	F	*p*<
A (100)	1.06 (.136)	.68 (.172)	.65 (.233)	2.24 (.336)	(2,1541) = 12.995	.0005
B (1000)	-.23 (.162)	-.39 (.175)	-.64 (.364)	.61 (.338)	(2, 1522) = 4.503	.05
C (723)	4.34 (.230)	1.17 (.222)	3.81 (.373)	10.49 (.557)	(2, 1311) = 163.245	.0005

Table 23. Directional Errors by Estimator Groups ANOVA

Estimator Groups' Directional Error ANOVA			
Block A	**Good (N=26)**	**Average (N=19)**	**Poor (N=15)**
Good			p<.0005
Average			p<.0005
Poor	p<.0005	p<.0005	
Block B	**Good (N=26)**	**Average (N=19)**	**Poor (N=15)**
Good			p<.05
Average			p<.05
Poor	p<.05	p<.05	
Block C	**Good (N=26)**	**Average (N=19)**	**Poor (N=15)**
Good		p<.0005	p<.0005
Average	p<.0005		p<.0005
Poor	p<.0005	p<.0005	

Table 24. Estimator Groups' Directional Errors by Denomination by Spectrum

Estimator Groups' Directional Error ANOVA						
	Good	F	Average	F	Poor	F
Block A	M (SE)		M (SE)		M (SE)	
Lower	3.00 (.217)	(1, 614) = 201.567	3.01 (.289)	(1, 448) = 104.402	5.36 (.452)	(1, 357) = 86.624
Upper	-1.54 (.234)	p<.0005	-1.62 (.347)	p<.0005	-.71 (.470)	p<.0005
Block B						
Lower	2.02 (.228)	(1, 619) = 224.976	1.86 (.489)	(1, 449) = 44.200	3.28 (.465)	(1, 334) = 63.504
Upper	-2.81 (.226)	p<.0005	-2.92 (.528)	p<.0005	-2.06 (.481)	p<.0005
Block C						
Lower	1.98 (.286)	(1, 516) = 8.926	4.71 (.516)	(1, 376) = 2.995	10.70 (.934)	(1, 298) = .511
Upper	.57 (.376)	p<.005	3.35 (.589)	NS	9.85 (.741)	NS

Table 25. Fixations per blocks

Blocks	Mark	Start	Left Middle	Middle	Right Middle	End	Total Fixations
	M (SE)	M (SE)	M (SE)	M (SE)	M (SE)	M (SE)	M (SE)
A (100)	1.74 (.085)	.90 (.061)	1.70 (.089)	2.20 (.083)	1.68 (.079)	2.19 (.071)	8.67 (.188)
B (1000)	1.73 (.058)	.74 (.074)	1.78 (.125)	2.46 (.100)	2.12 (.112)	2.68 (.129)	9.78 (.306)
C (723)	1.86 (.068	.71 (.039)	1.53 (.072)	2.36 (.081)	1.80 (.098)	1.98 (.074)	8.36 (.182)

Table 26. Fixation per block by spectrum

Blocks	Mark	Start	Left Middle	Middle	Right Middle	End	Total Fixation s	F
A (100)								
Lower	1.76 (.141)	1.62 (.121)	2.99 (.173)	2.27 (.136)	.75 (.057)	1.40 (.069)	9.03 (.316)	NS
Upper	1.71 (.102)	.27 (.035)	.51 (.038)	1.88 (.099)	2.69 (.148)	3.07 (.125)	8.42 (.238)	
B (1000)								
Lower	1.94 (.093)	1.31 (.135)	2.84 (.265)	2.52 (.192)	1.55 (.202)	2.00 (.126)	10.20 (.598)	NS
Upper	1.57 (.081)	.34 (.097)	.84 (.115)	2.22 (.109)	2.77 (.148)	3.34 (.218)	9.50 (.368)	
C (723)								
Lower	2.08 (.111)	1.00 (.060)	2.11 (.118)	2.49 (.114)	1.20 (.141)	1.42 (.068)	8.21 (.256)	NS
Upper	1.68 (.094)	.41 (.049)	.85 (.064)	1.85 (.099)	2.56 (.144)	2.80 (.156)	8.47 (.275)	

Table 27. AOI Fixations for all three blocks by Estimator Groups

	Areas of Interest					
Estimator Groups	Start, M (SE)	Left Middle	Middle	Right Middle	End	Total, M (SE)
Good	0.81 (.050)	1.78 (.089)	2.64 (.079)	2.26 (.100)	2.56 (.097)	10.06 (.225)
Average	0.81 (.071)	1.56 (.104)	2.13 (.086	1.48 (.071	2.05 (.076)	8.03 (.203)
Poor	0.71 (0.50)	1.57 (.081)	2.03 (.099)	1.59 (.098	1.97 (.079	7.88 (.205)

Table 28. AOI Fixations for Block A (100) by Estimator Group

Estimator Groups	Areas of Interest					
	Start, M (SE)	Left Middle	Middle	Right Middle	End	Total, M (SE)
Good	0.94 (.087)	1.68 (.109)	2.22 (.095)	1.83 (.104)	2.34 (.103)	9.01 (.224)
Average	0.84 (.131)	1.78 (.228)	2.20 (.164)	1.43 (.128)	2.01 (.134)	8.27 (.425)
Poor	0.89 (.106)	1.63 (.131)	2.14 (.208)	1.70 (.198)	2.13 (.141)	8.49 (.392)

Table 29. AOI Fixations for Block C (723) by Estimator Group

Estimator Groups	Areas of Interest					
	Start, M (SE)	Left Middle	Middle	Right Middle	End	Total, M (SE)
Good	0.75 (.061	1.73 (.123)	2.78 (.138)	2.30 (.192)	2.12 (.126)	9.69 (.316)
Average	0.64 (.061)	1.25 (.093)	2.03 (.131)	1.42 (.115)	1.95 (.122)	7.29 (.255)
Poor	0.72 (.080)	1.52 (.143)	2.00 (.133)	1.37 (.123)	1.75 (.127)	7.35 (.313)

Table 30. Last Fixation by AOI by Block

	AOI, Frequency (Percentage)					
	Mark	Start	Left Middle	Middle	Right Middle	End
Block A (100)	490 (50.2)	49 (5.0)	62 (6.4)	78 (8.0)	93 (9.5)	156 (16.0)
Block B (1000)	475 (49.1)	45 (4.6)	51 (5.3)	77 (8.0)	101 (10.4)	168 (17.4)
Block C (723)	293 (36.6)	32 (4.0)	51 (6.4)	97 (12.1)	113 (14.1)	176 (22.0)

Table 31. Last Fixation by AOI by Math Anxiety Group, Block A (100)

	AOI, Frequency (Percentage)					
Block A	Mark	Start	Left Middle	Middle	Right Middle	End
Low MANX	166 (50.6)	17 (5.2)	16 (4.9)	24 (7.30)	34 (10.4)	54 (16.5)
Medium MANX	98 (45.8)	6 (2.8)	19 (8.9)	20 (9.3)	31 (14.5)	34 (15.9)
High MANX	90 (48.6)	13 (7)	13 (7)	15 (8.1)	15 (8.1)	25 (13.5)

Table 32. Last Fixation by AOI by Math Anxiety Group, Block B (1000)

	AOI, Frequency (Percentage)					
Block B	Mark	Start	Left Middle	Middle	Right Middle	End
Low MANX	147 (48)	17 (5.6)	14 (4.6)	25 (8.2)	29 (9.5)	56 (18.3)
Medium MANX	108 (43.7)	8 (3.20	18 (7.3)	25 (10.1)	30 (12.1)	49 (19.6)
High MANX	93 (53.8)	8 (4.6)	7 (4.0)	6 (3.5)	18 (10.4)	34 (19.7)

Table 33. Last Fixation by AOI by Math Anxiety Group, Block C (723)

	AOI, *Frequency (Percentage)*					
Block C	Mark	Start	Left Middle	Middle	Right Middle	End
Good Estimators	76 (31.0)	7 (2.9)	20 (8.2)	33 (13.5)	53 (21.6)	37 (15.1)
Average Estimators	62 (29.7)	12 (5.7)	13 (6.2)	32 (15.3)	26 (12.4)	55 (26.3)
Poor Estimators	69 (45.4)	6 (3.9)	7 (4.6)	18 (11.8)	19 (12.5)	30 (19.7)

Table 34. Last Fixation by AOI by Estimator Group

	Last Fixation by AOI, Frequency (percentage)					
Block A	Mark	Start	Left Middle	Middle	Right Middle	End
Good Estimators	176 (39.1)	22 (4.9)	27 (6.0)	40 (8.9)	58 (12.9)	98 (21.8)
Average Estimators	154 (58.1)	11 (4.2)	21 (7.9)	21 (7.9)	19 (7.2)	33 (12.5)
Poor Estimators	160 (61.3)	16 (6.1)	14 (5.4)	17 (6.5)	16 (6.1)	25 (9.6)
Block B						
Good Estimators	216 (47.7)	17 (3.8)	24 (5.3)	44 (9.7)	45 (9.9)	86 (18.8)
Average Estimators	121 (42.2)	19 (6.6)	22 (7.7)	21 (7.3)	36 (12.5)	48 (16.7)
Poor Estimators	138 (60.5)	9 (3.9)	5 (2.2)	12 (5.3)	20 (8.8)	35 (15.4)
Block C						
Good Estimators	126 (33)	12 (3.1)	25 (6.5)	53 (13.9)	59 (15.4)	92 (24.1)
Average Estimators	88 (37.4)	12 (5.1)	17 (7.2)	23 (9.8)	33 (14.0)	47 (20)
Poor Estimators	79 (43.2)	8 (4.4)	9 (4.9)	21 (11.5)	21 (11.5)	37 (20.2)

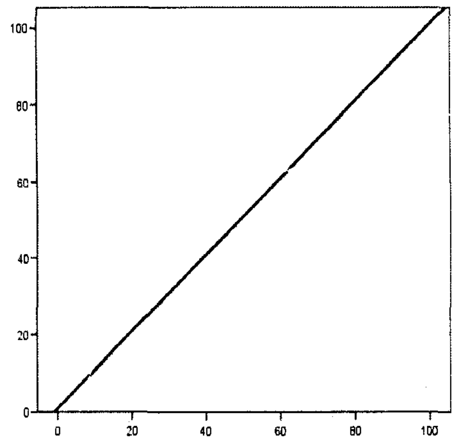

Figure 1. Graph of a linear Function

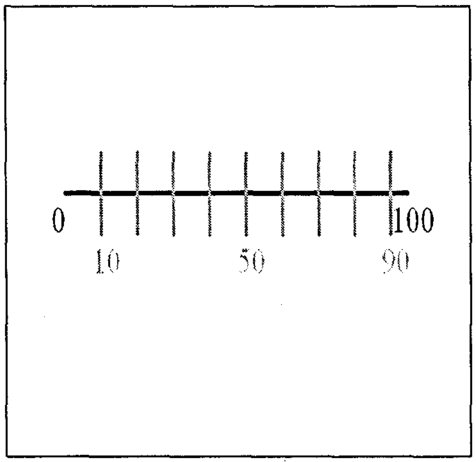

Figure 2. Overlay of a Linear Number Line

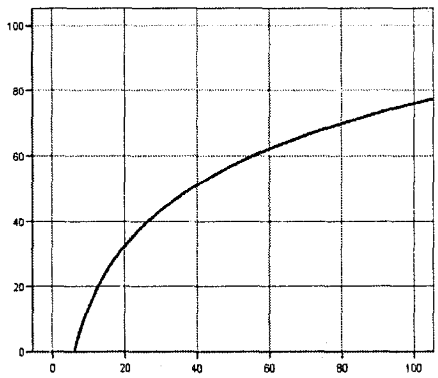

Figure 3. Graph of a Logarithmic Function

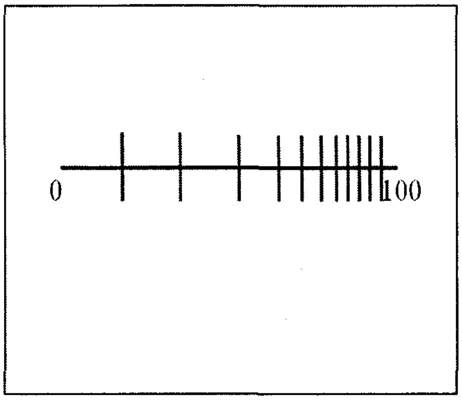

Figure 4. Overlay of a Logarithmic Number Line

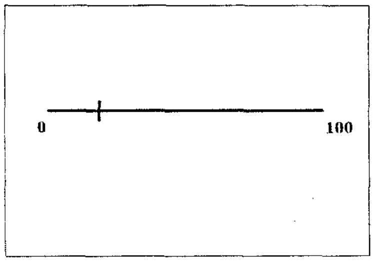

Figure 5. Example of stimuli, 0 to 100 number line (18).

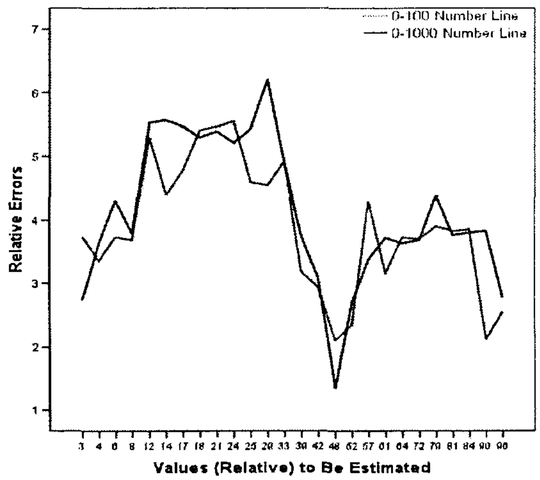

Figure 6. Graph of Relative Errors, 100 and 1000 Number Lines

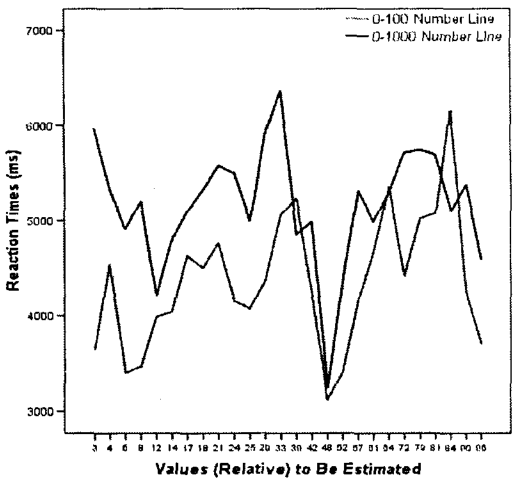

Figure 7. Graph of Reaction Times, 100 and 1000 Number Lines.

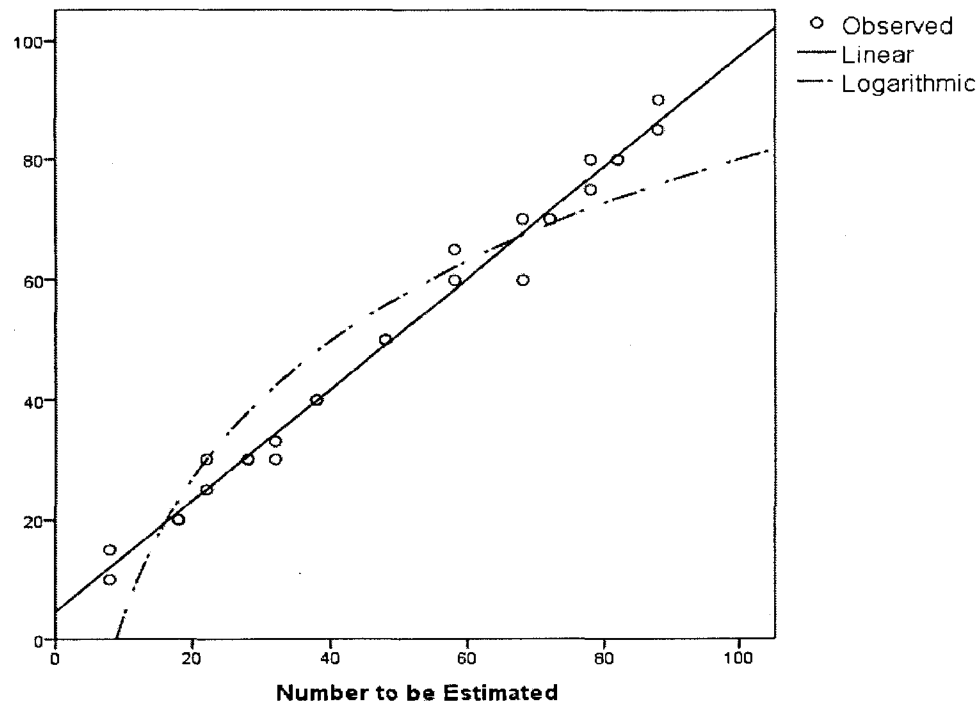

Figure 8. Example of a Linear Response

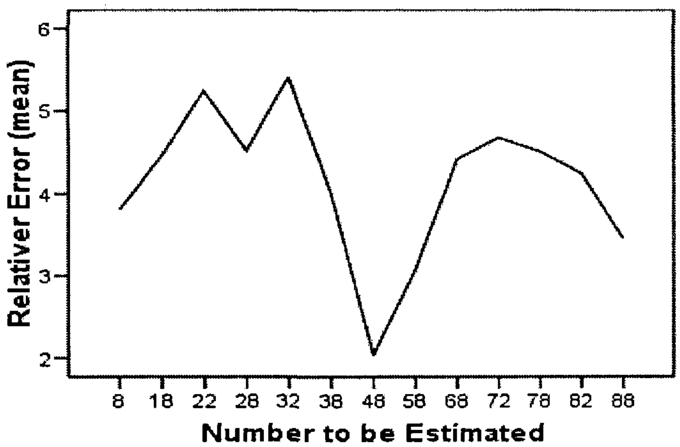

Figure 9. Relative Errors on 100 number lines (A)

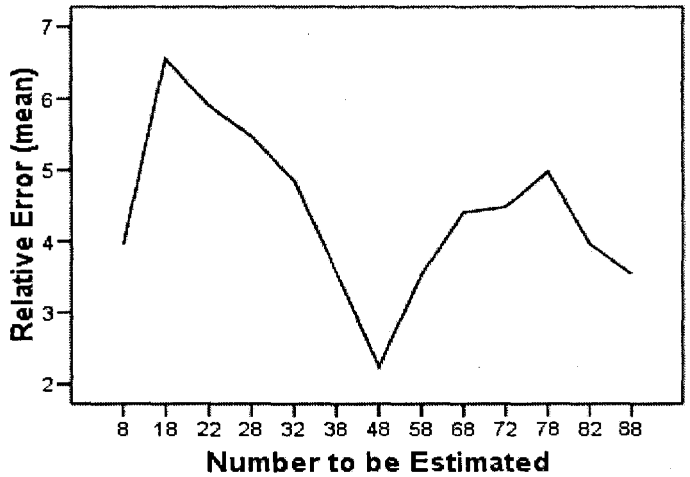

Figure 9. Relative Errors on 100 number lines (B)

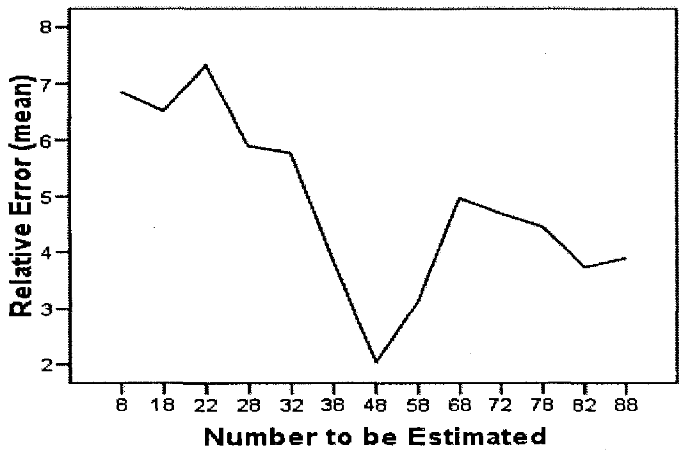

Figure 9. Relative Errors on 100 number lines (C)

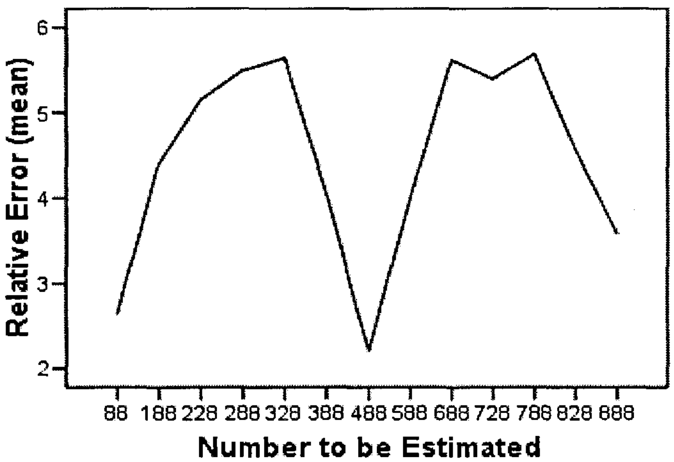

Figure 10. Relative Errors on 1,000 number lines (A)

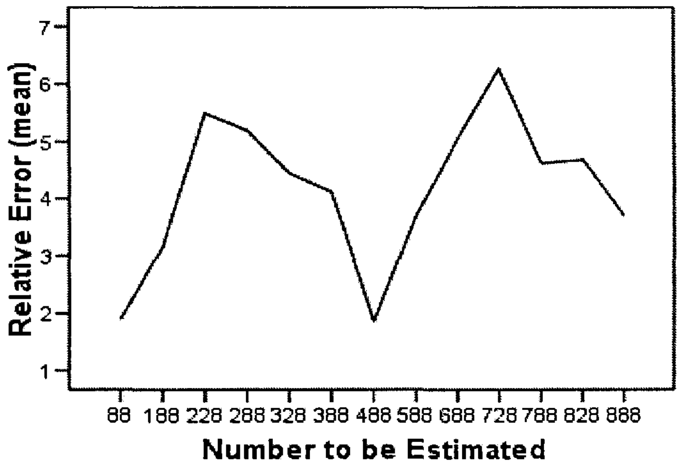

Figure 10. Relative Errors on 1,000 number lines (A)

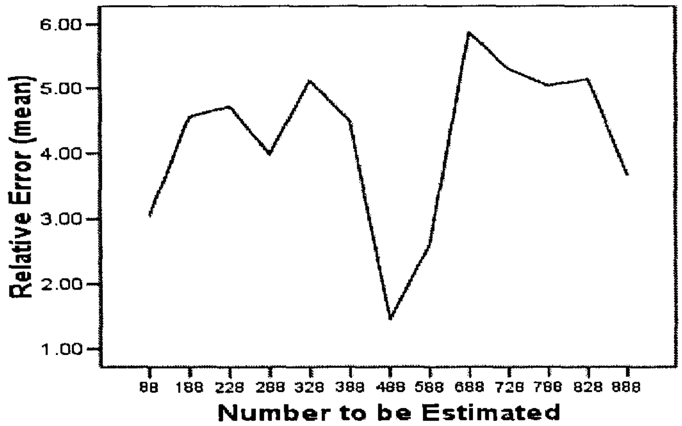

Figure 10. Relative Errors on 1,000 number lines (C)

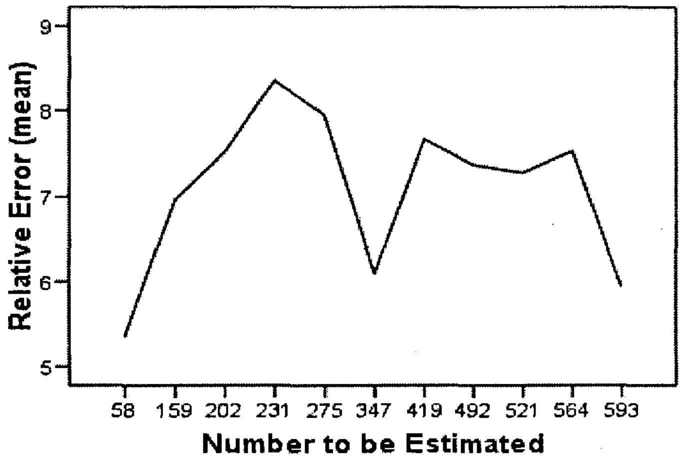

Figure 11. Relative Errors on 723 number lines (A)

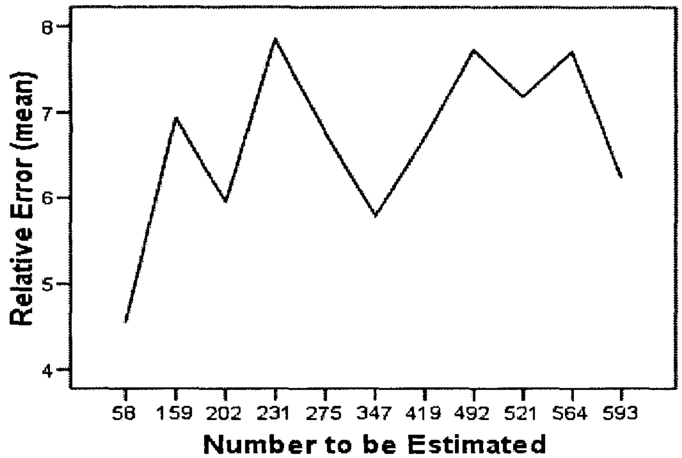

Figure 11. Relative Errors on 723 number lines (B)

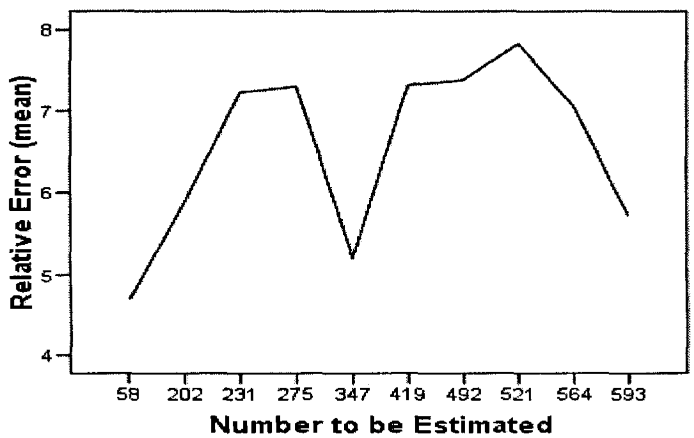

Figure 11. Relative Errors on 723 number lines (C)

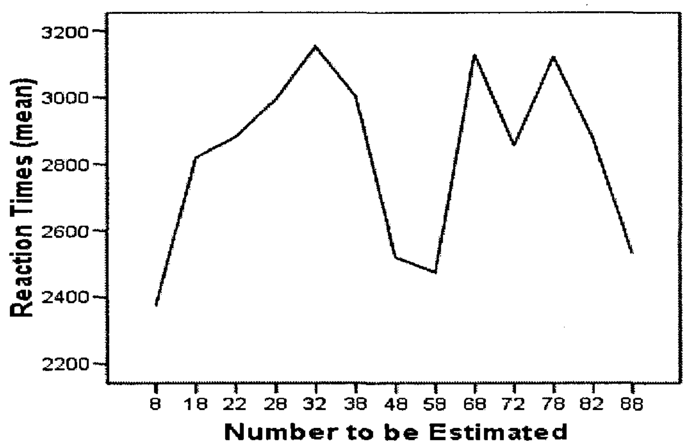

Figure 12. Reaction Time results on 100 number lines (A)

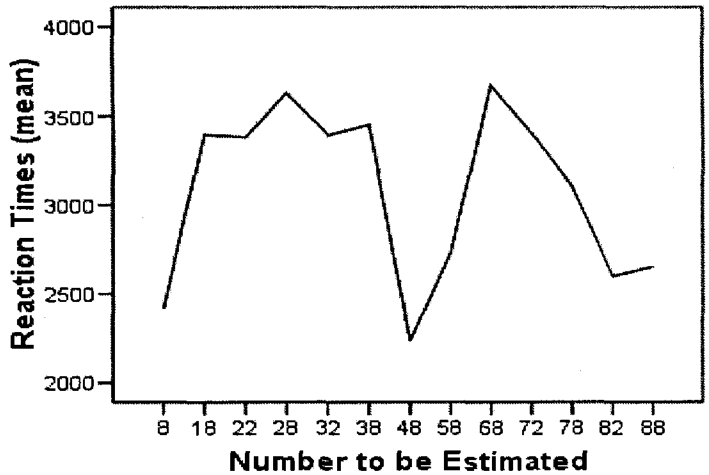

Figure 12. Reaction Time results on 100 number lines (B)

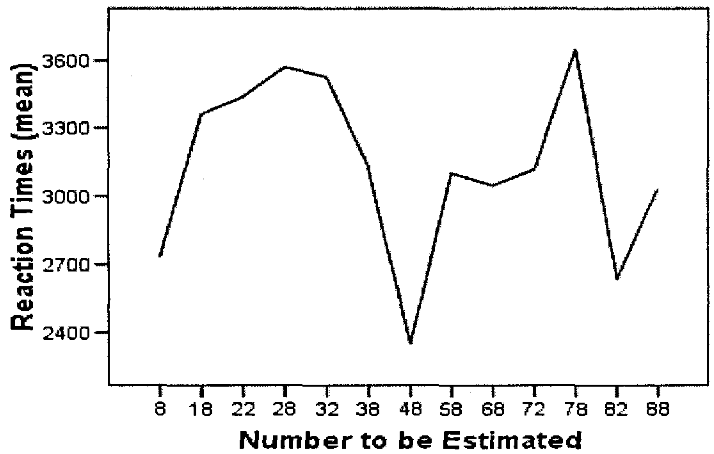

Figure 12. Reaction Time results on 100 number lines (C)

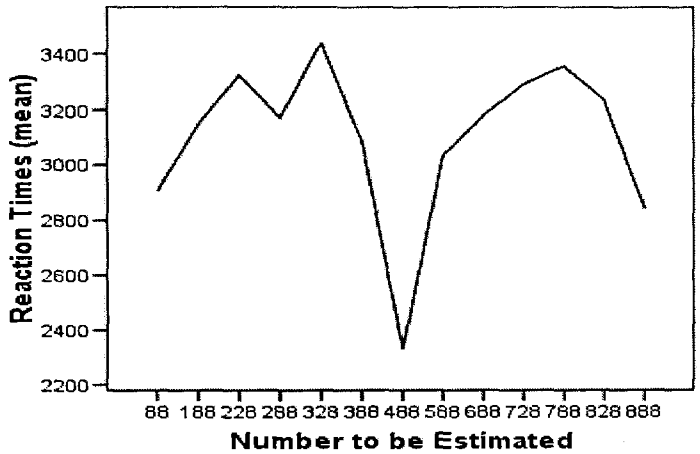

Figure 13. Reaction Time results on 1,000 number lines (A)

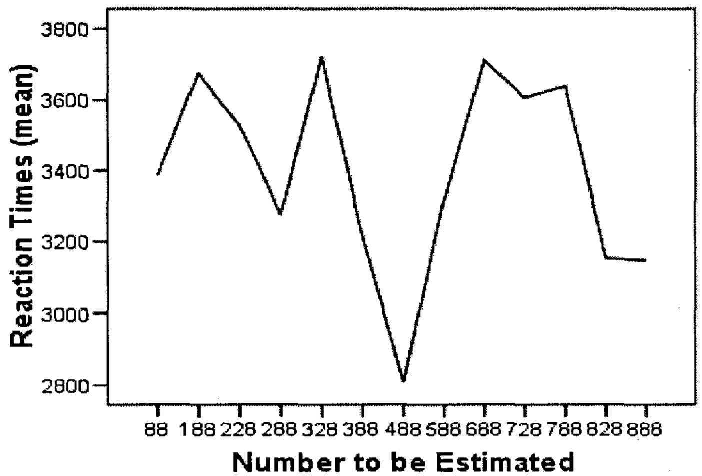

Figure 13. Reaction Time results on 1,000 number lines (B)

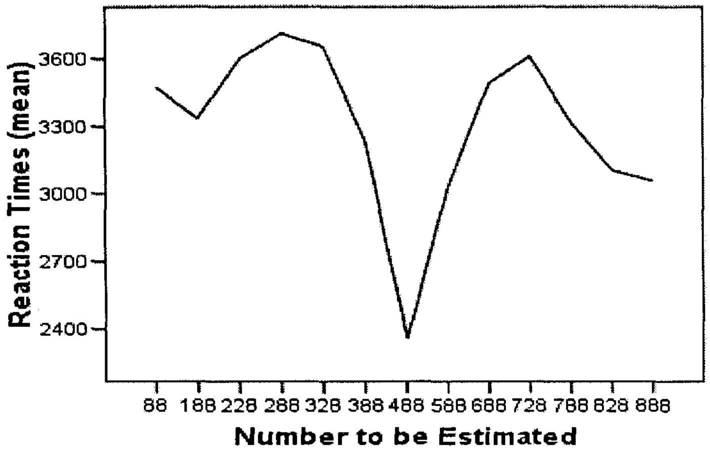

Figure 13. Reaction Time results on 1,000 number lines (C)

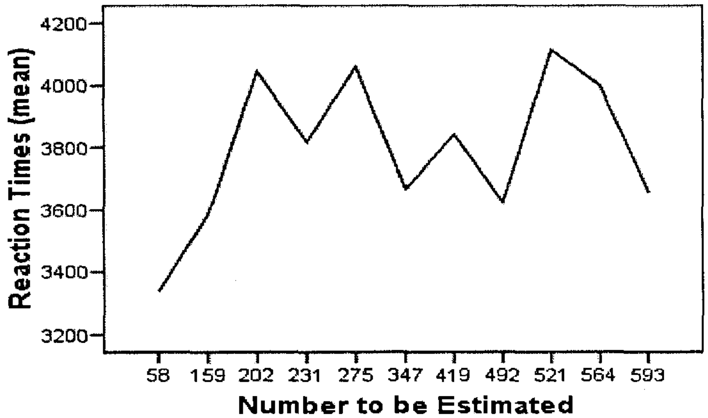

Figure 14. Reaction Time results on 723 number lines (A)

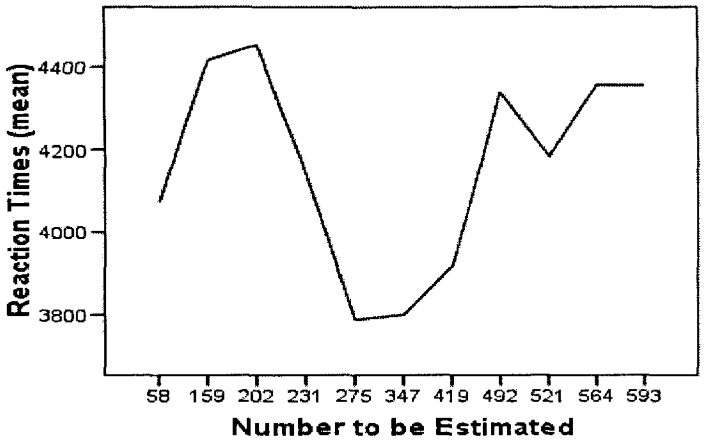

Figure 14. Reaction Time results on 723 number lines (B)

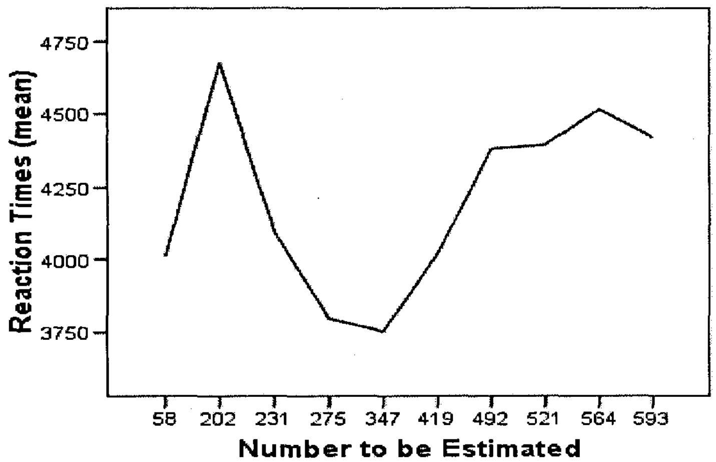

Figure 14. Reaction Time results on 723 number lines (C)

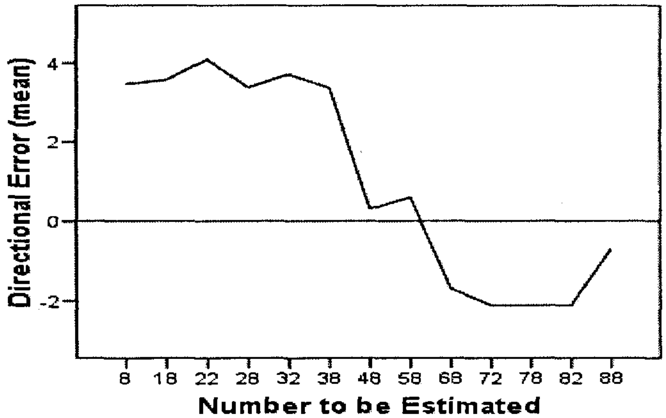

Figure 15. Directional Error on 100 number lines (A)

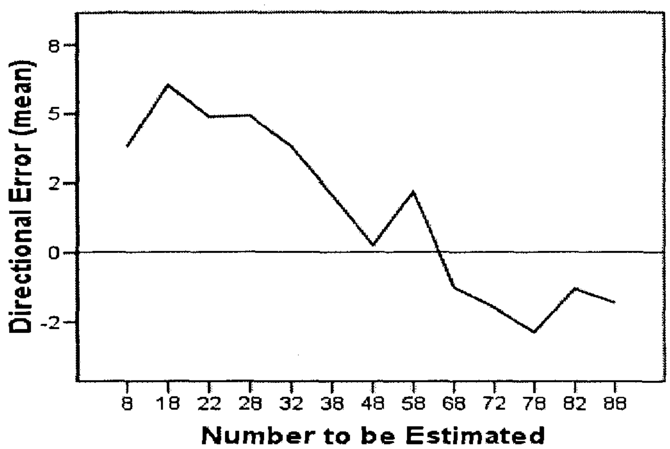

Figure 15. Directional Error on 100 number lines (B)

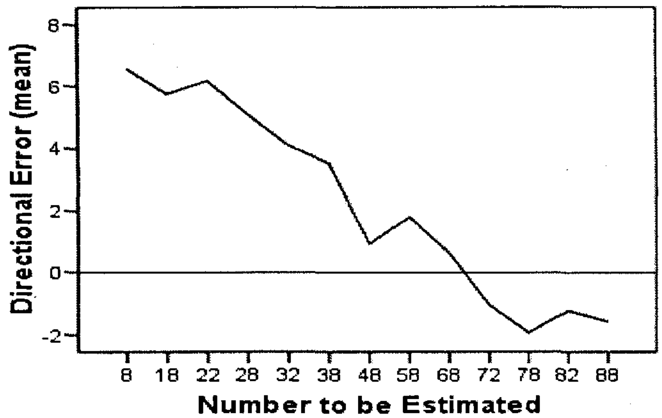

Figure 15. Directional Error on 100 number lines (C)

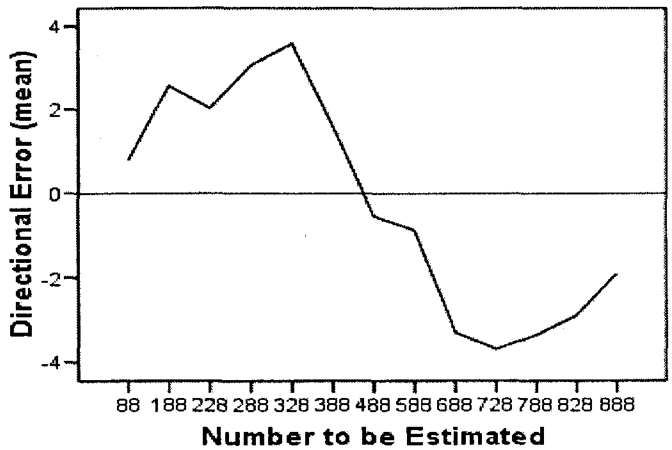

Figure 16. Directional Error on 1000 number lines (A)

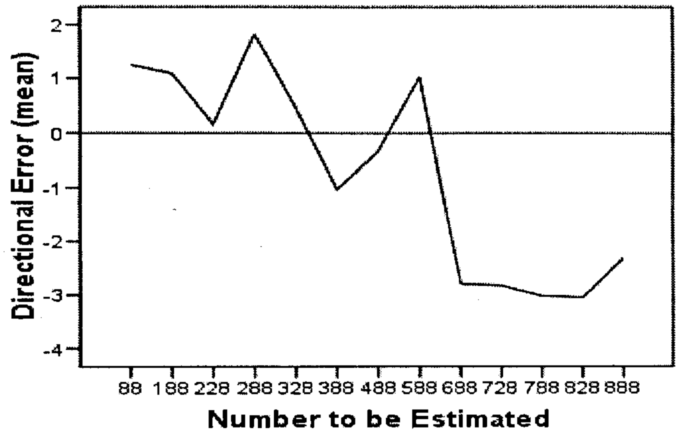

Figure 16. Directional Error on 1000 number lines (B)

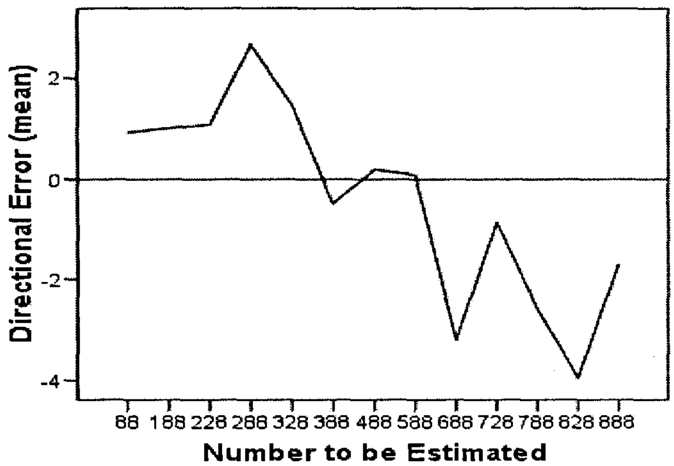

Figure 16. Directional Error on 1000 number lines (C)

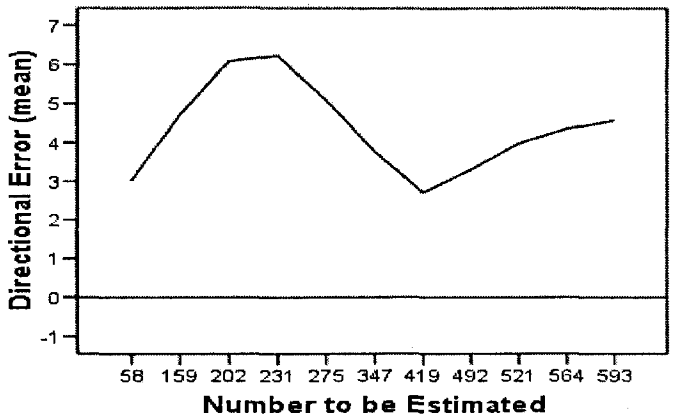

Figure 17. Directional Error on 723 number lines (A)

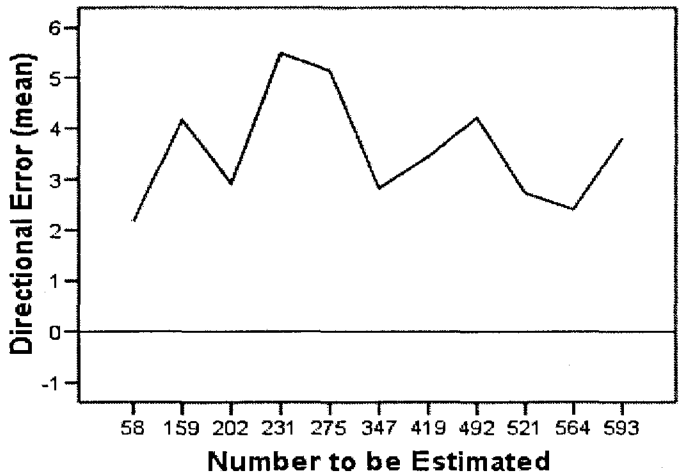

Figure 17. Directional Error on 723 number lines (B)

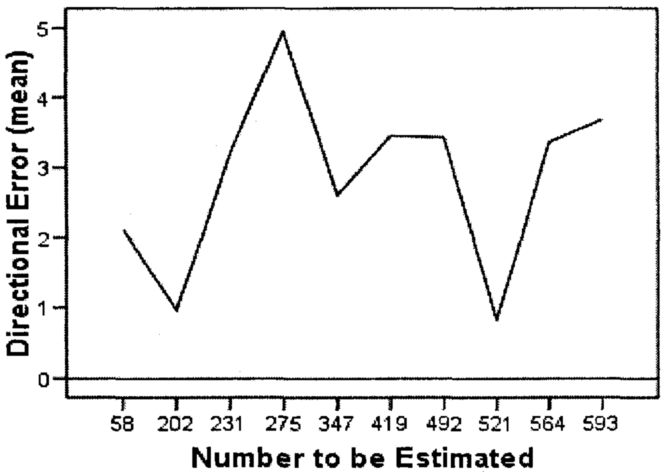

Figure 17. Directional Error on 723 number lines (C)

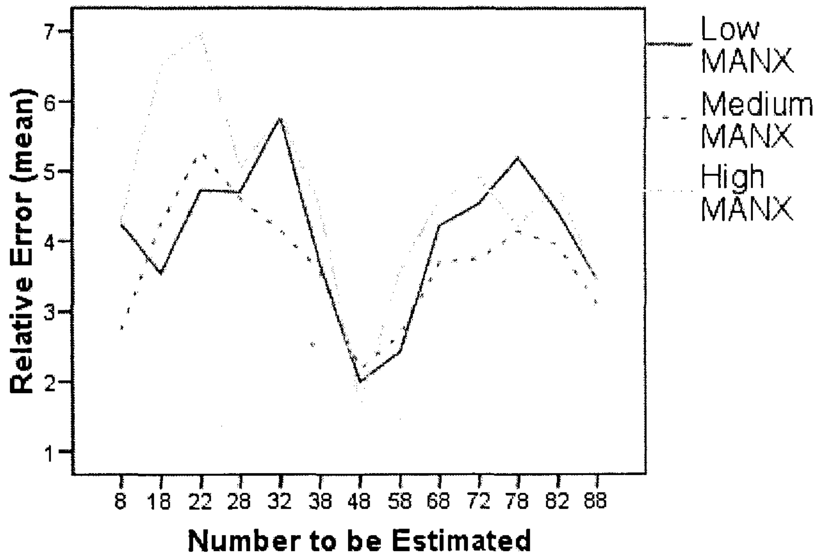

Figure 18. Relative Error by Math Anxiety Group (A)

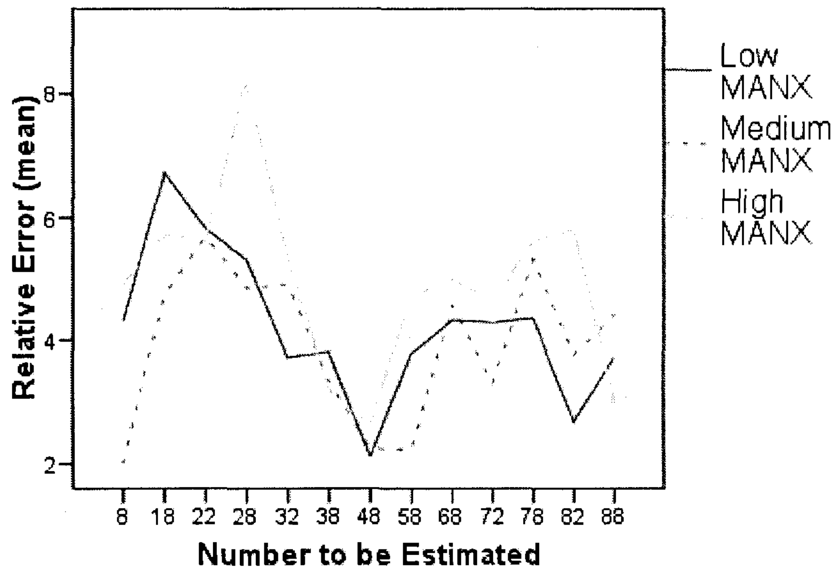

Figure 18. Relative Error by Math Anxiety Group (B)

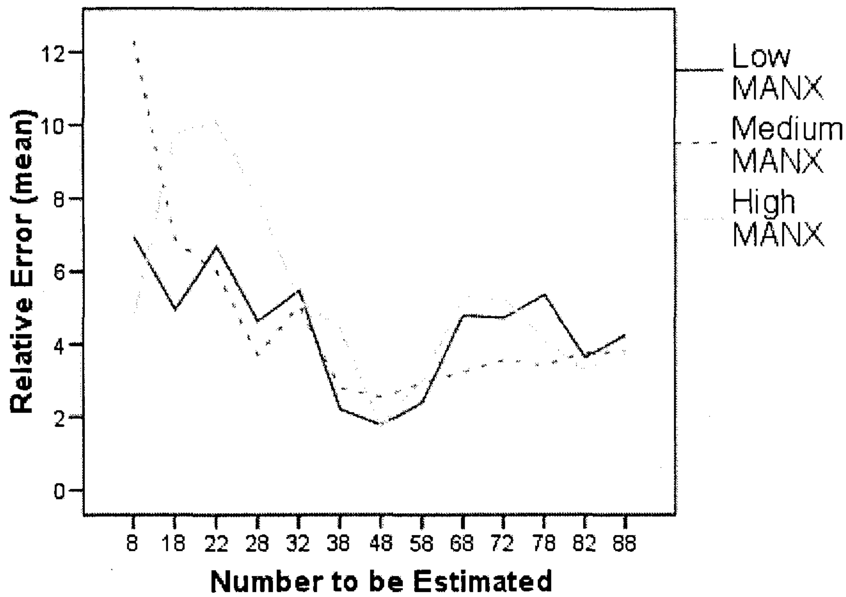

Figure 18. Relative Error by Math Anxiety Group (C)

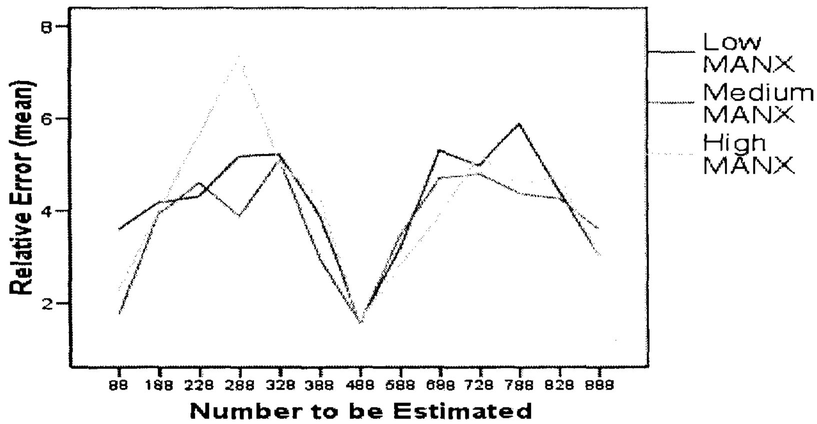

Figure 19. Relative Error by Math Anxiety Group (A)

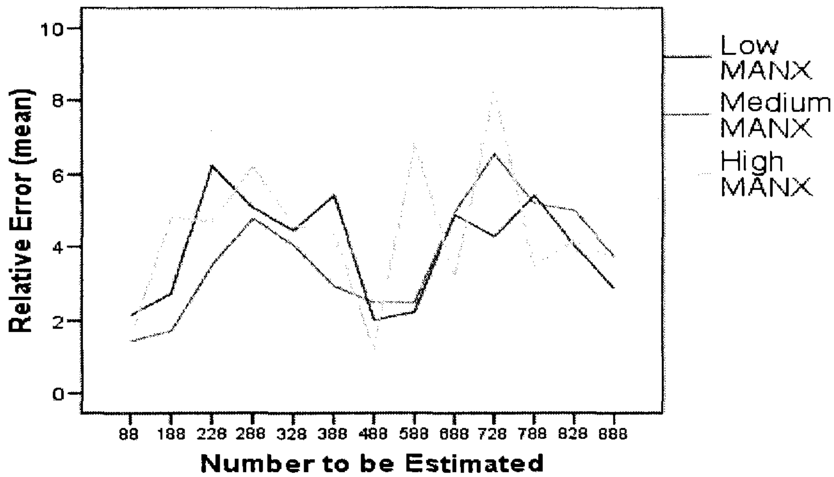

Figure 19. Relative Error by Math Anxiety Group (B)

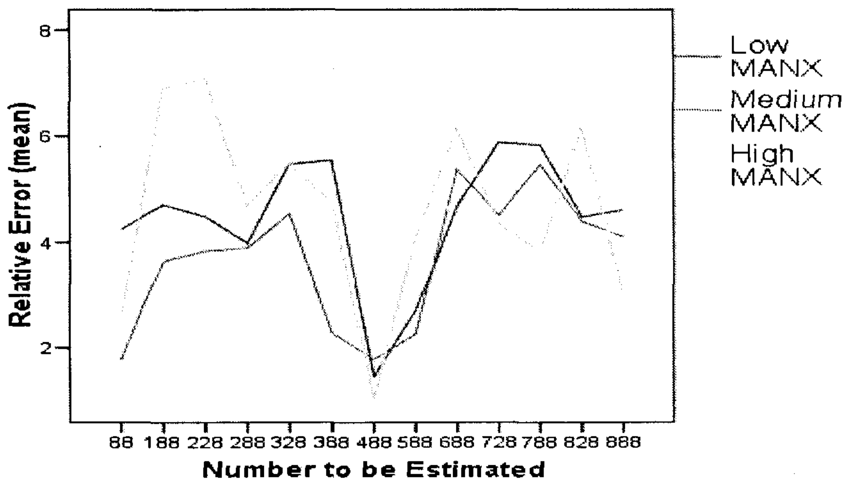

Figure 19. Relative Error by Math Anxiety Group (C)

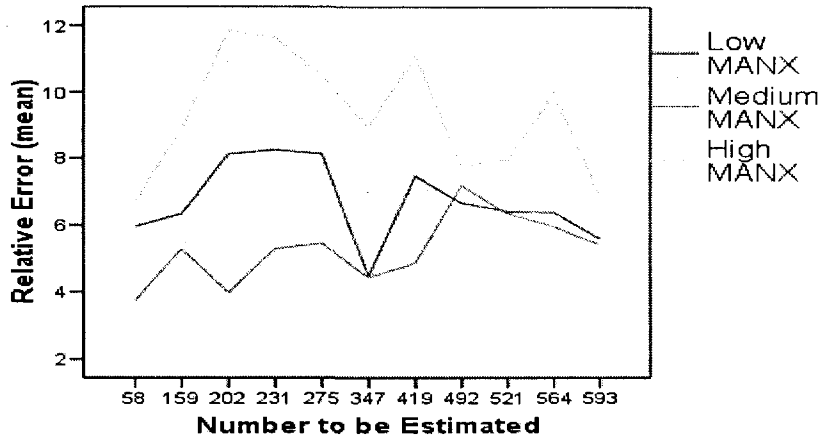

Figure 20. Relative Error by Math Anxiety Group (A)

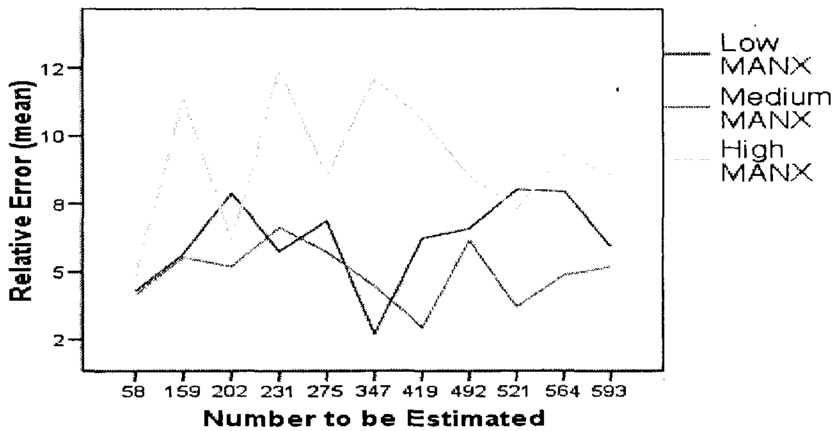

Figure 20. Relative Error by Math Anxiety Group (B)

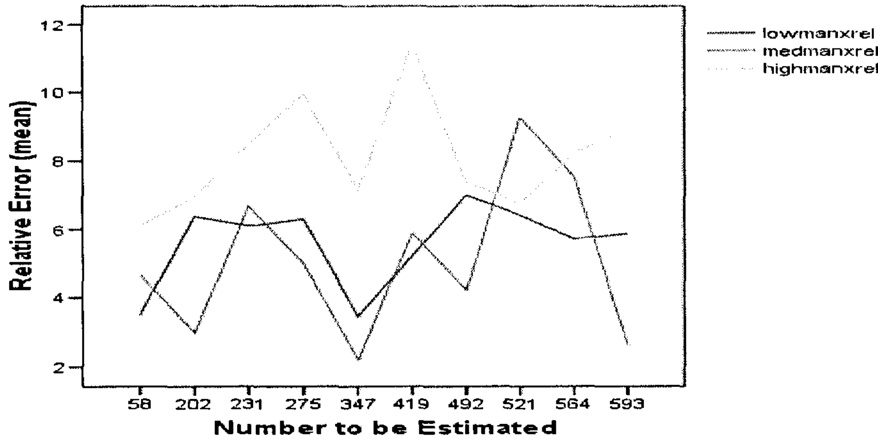

Figure 20. Relative Error by Math Anxiety Group (C)

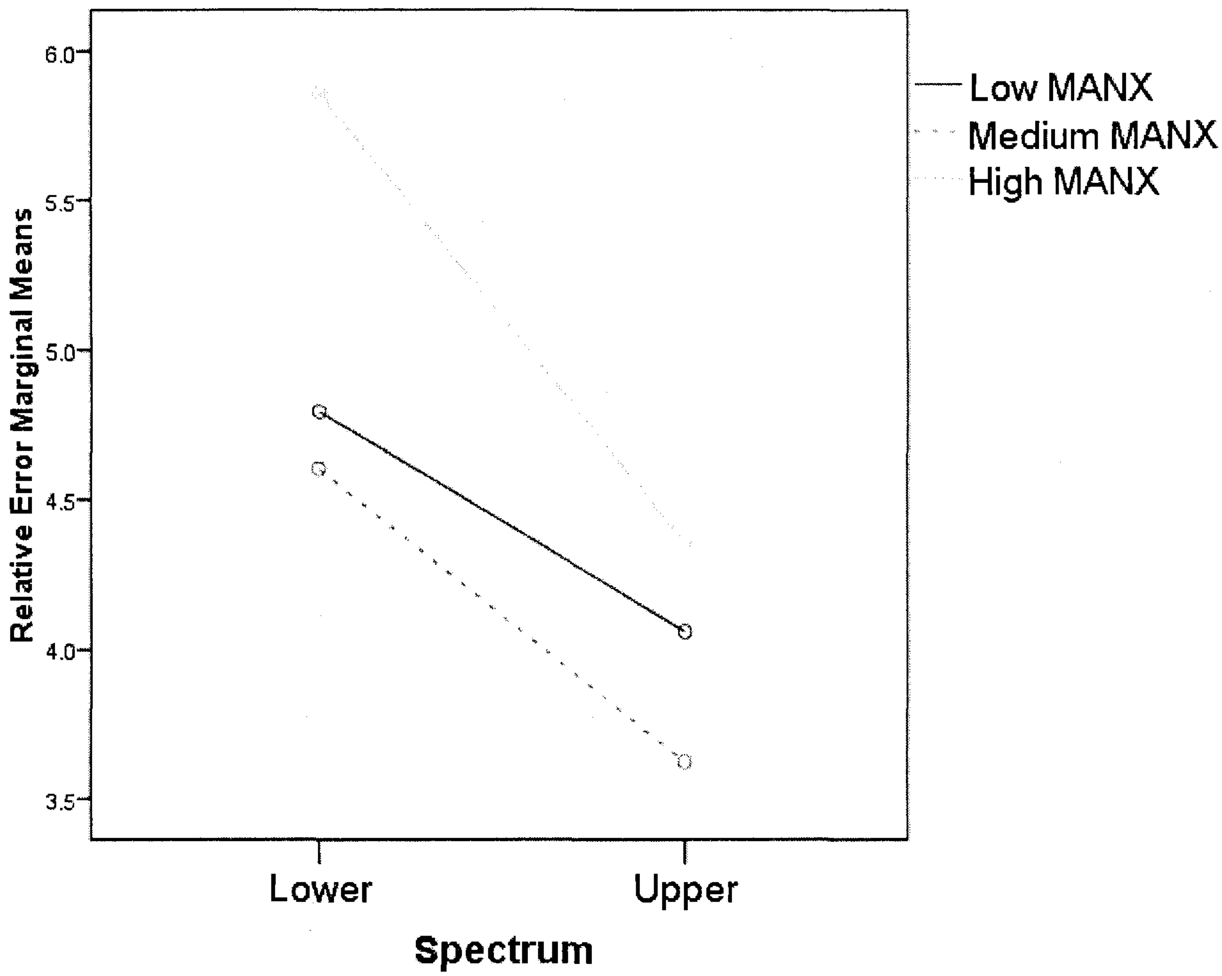

Figure 21. Relative Error by Math Anxiety Group by Spectrum, Block A (100)

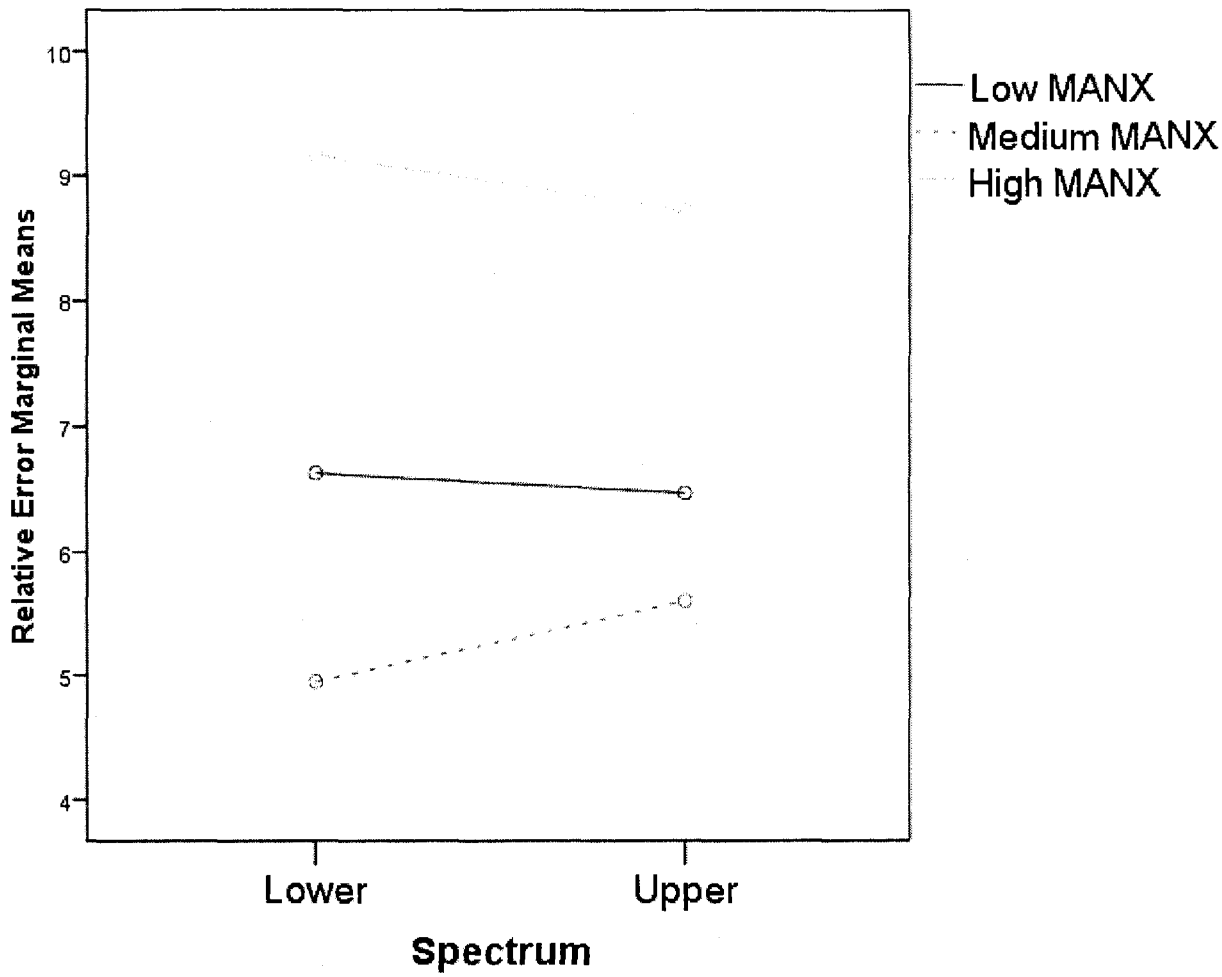

Figure 22. Relative Error by Math Anxiety Group by Spectrum, Block C (723)

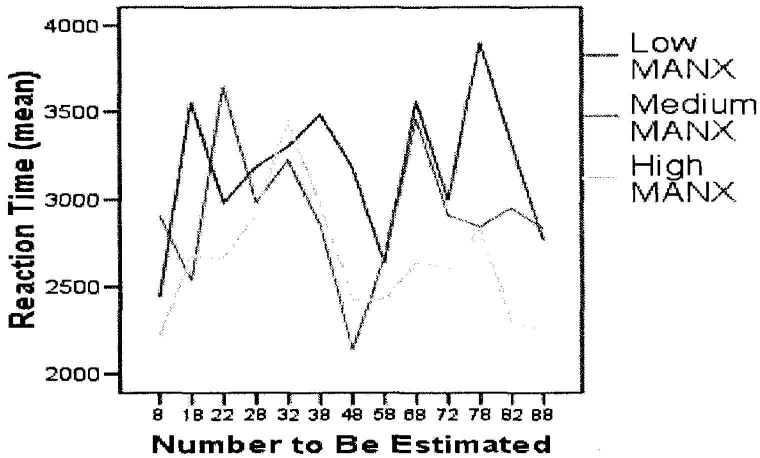

Figure 23. Reaction Time by Math Anxiety Group (A)

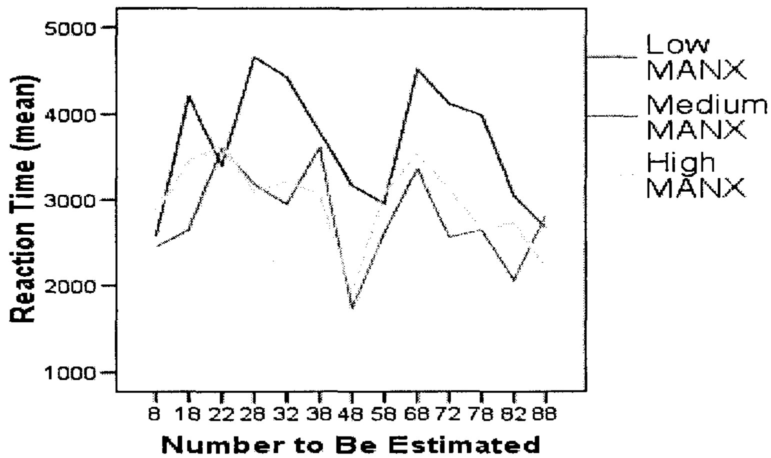

Figure 23. Reaction Time by Math Anxiety Group (B)

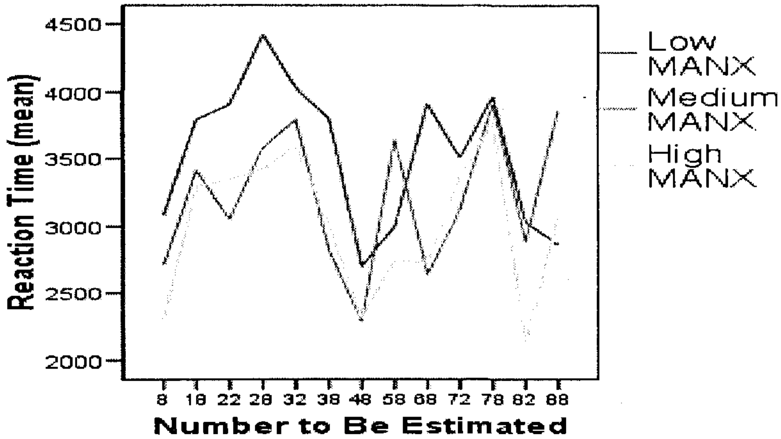

Figure 23. Reaction Time by Math Anxiety Group (C)

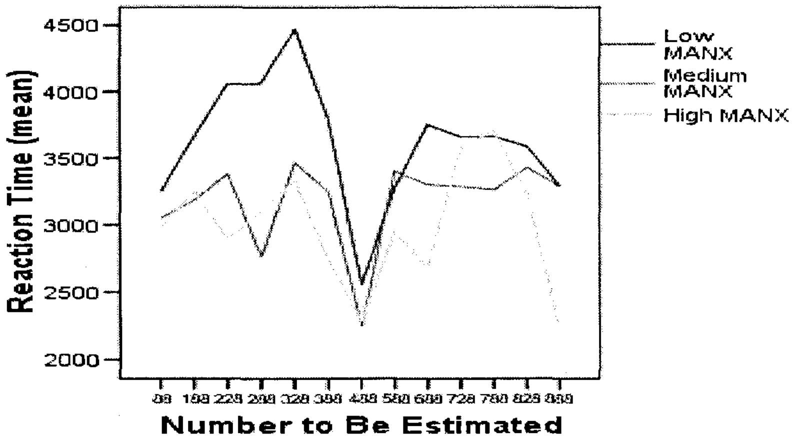

Figure 24. Reaction Time by Math Anxiety Group (A)

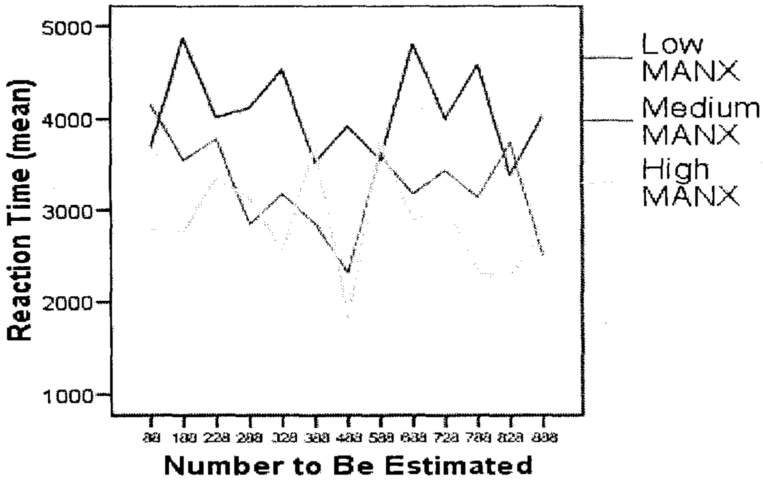

Figure 24. Reaction Time by Math Anxiety Group (B)

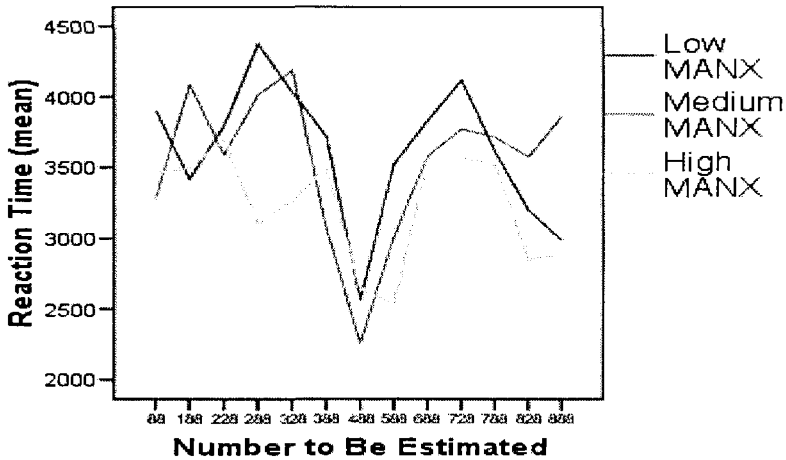

Figure 24. Reaction Time by Math Anxiety Group (C)

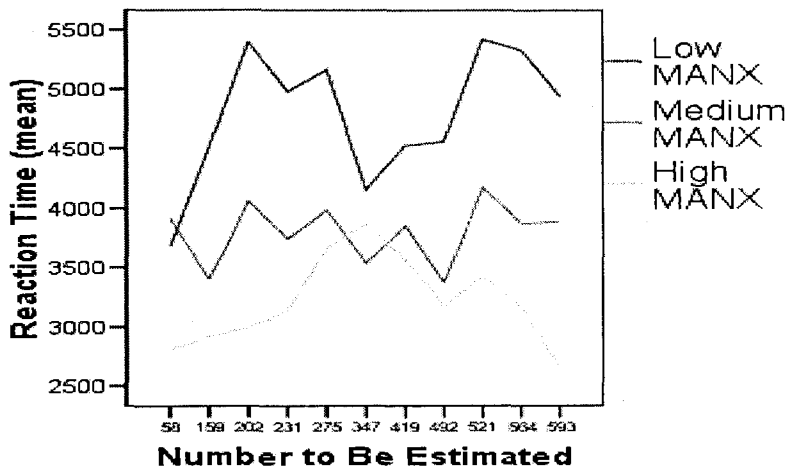

Figure 25. Reaction Time by Math Anxiety Group (A)

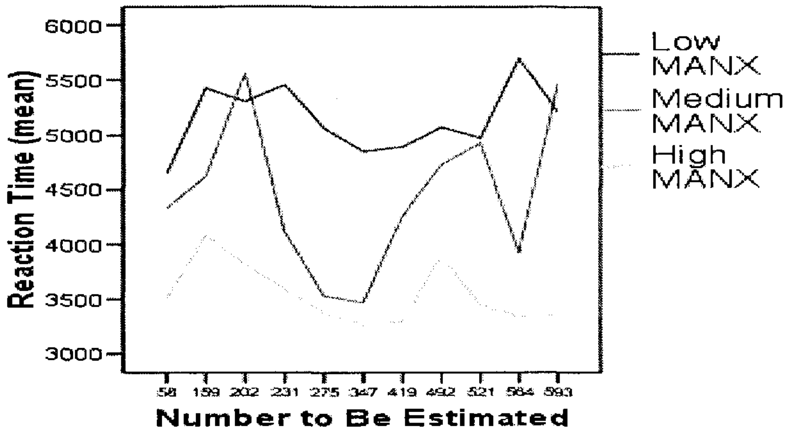

Figure 25. Reaction Time by Math Anxiety Group (B)

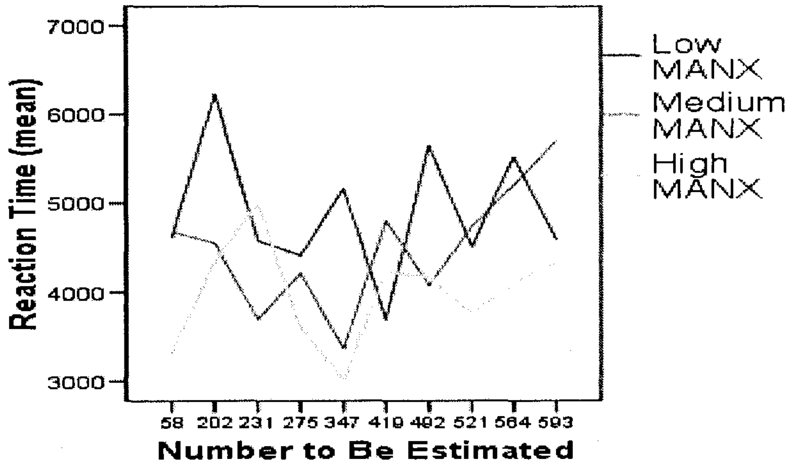

Figure 25. Reaction Time by Math Anxiety Group (C)

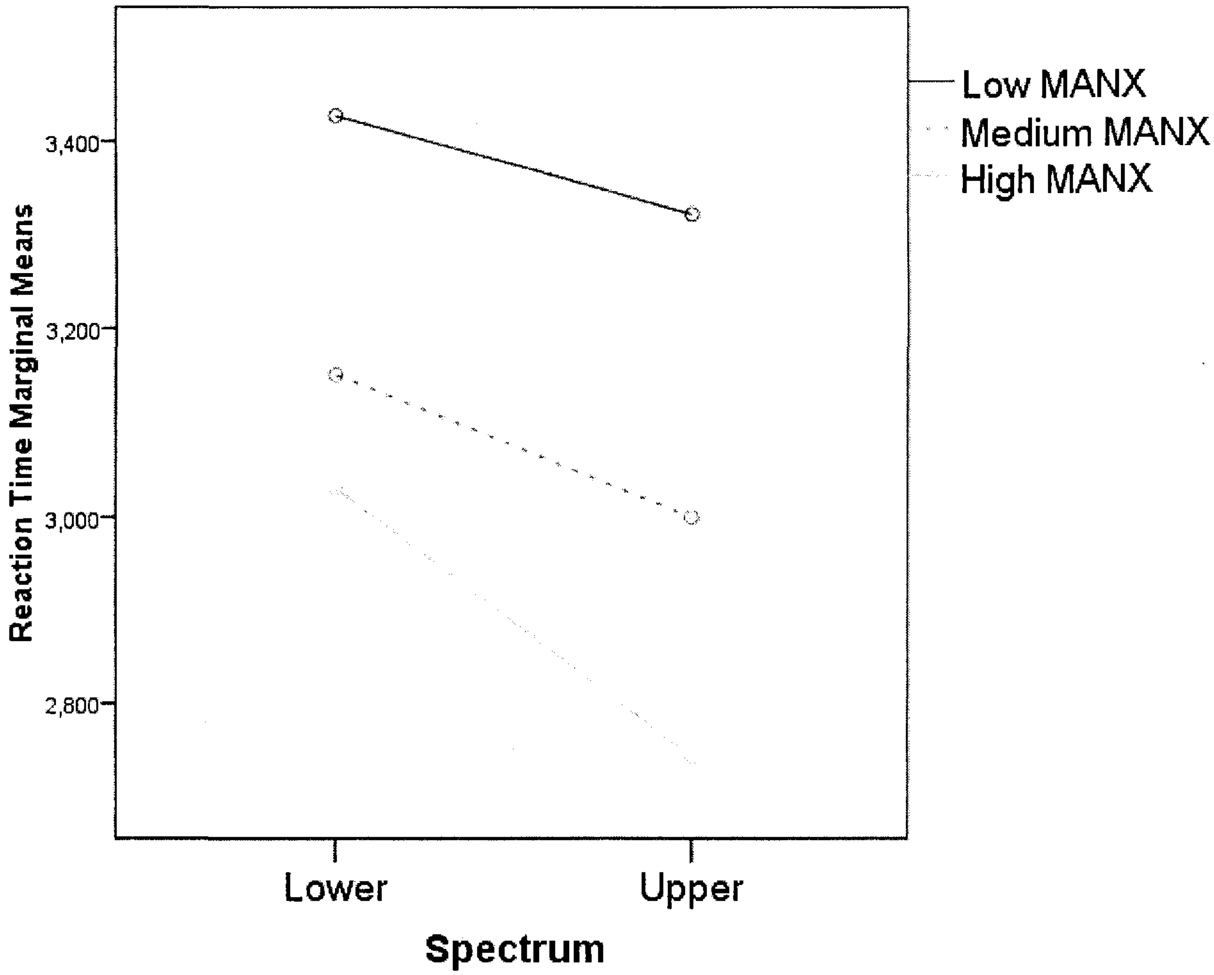

Figure 26. Reaction Time by Math Anxiety Group by Spectrum, Block A (100)

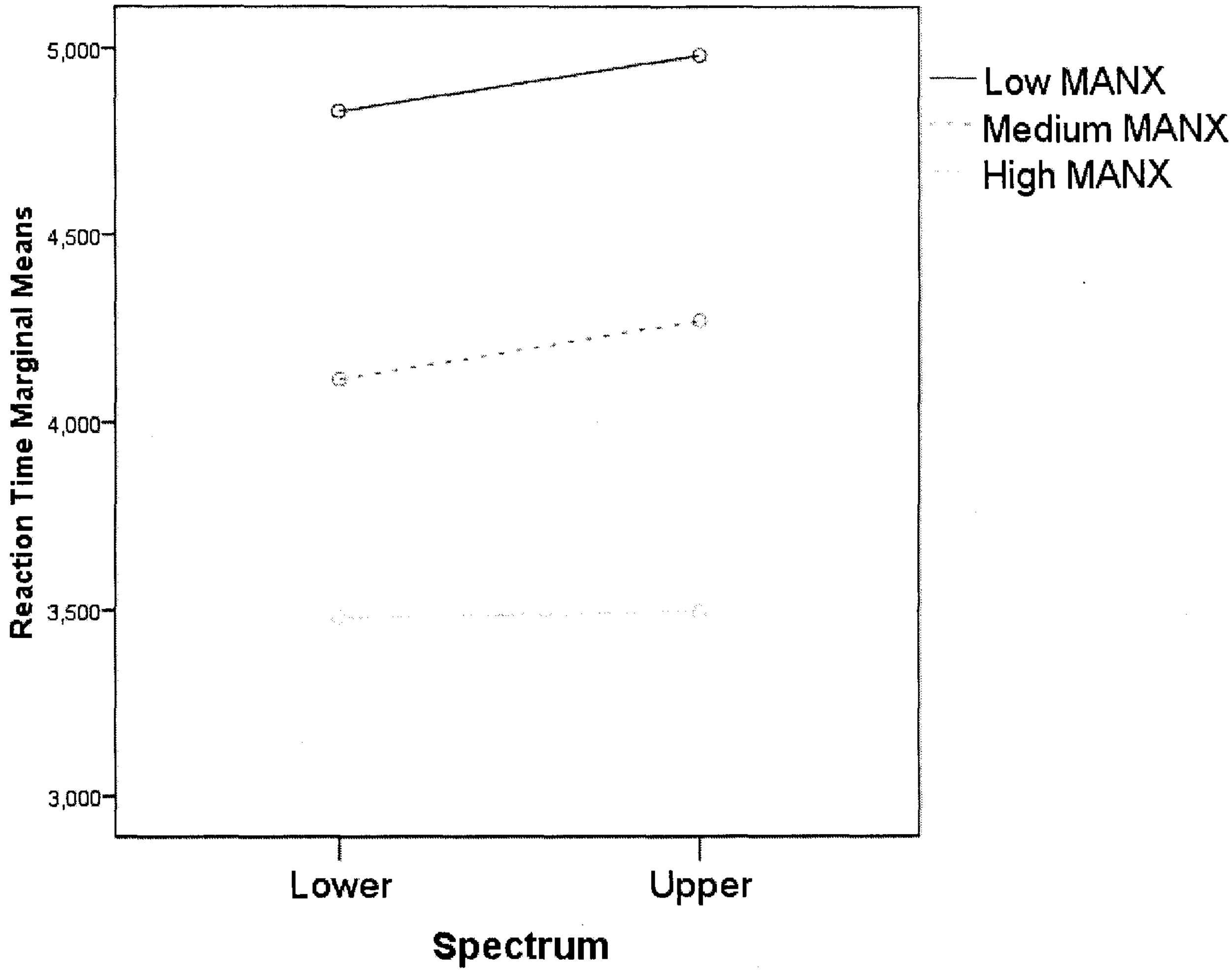

Figure 27. Reaction Time by Math Anxiety Group by Spectrum, Block C (723)

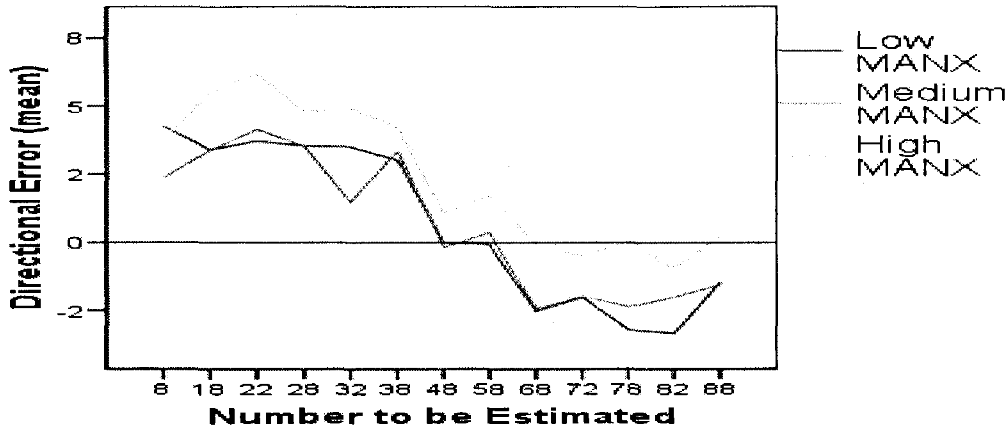

Figure 28. Directional Error by Math Anxiety Group (A)

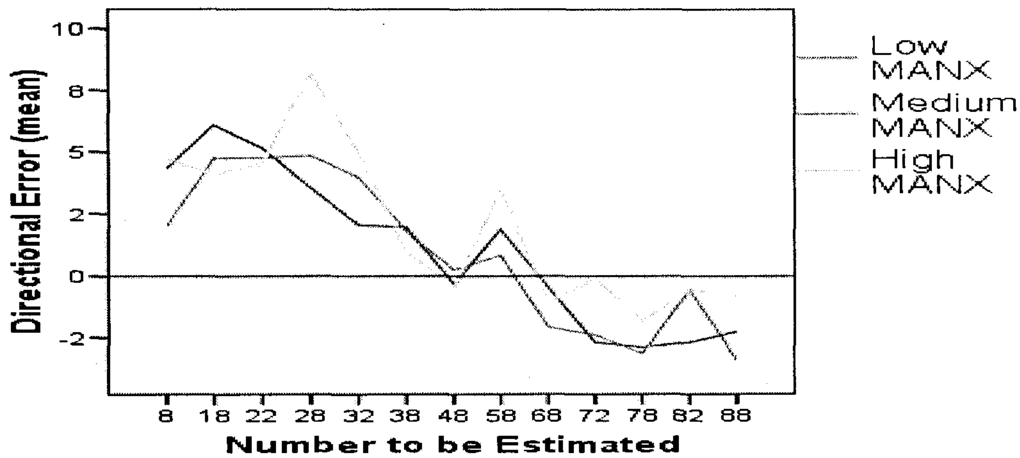

Figure 28. Directional Error by Math Anxiety Group (B)

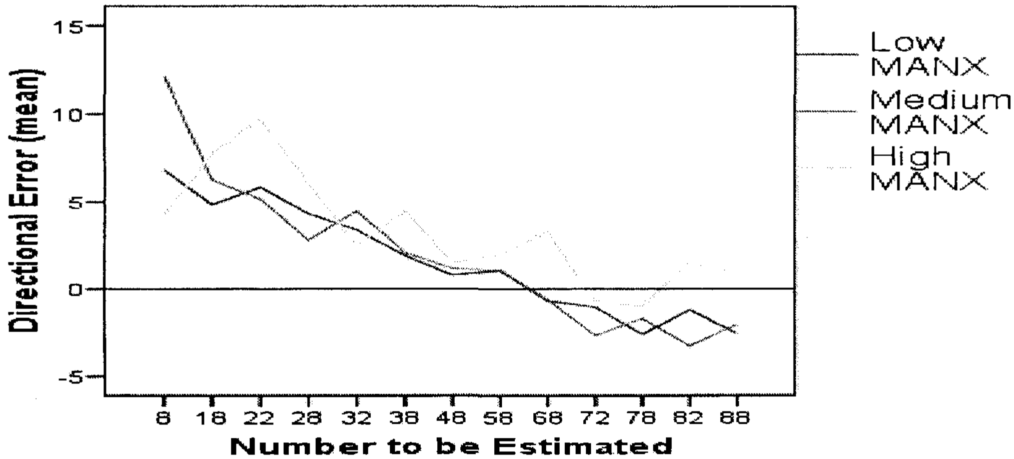

Figure 28. Directional Error by Math Anxiety Group (C)

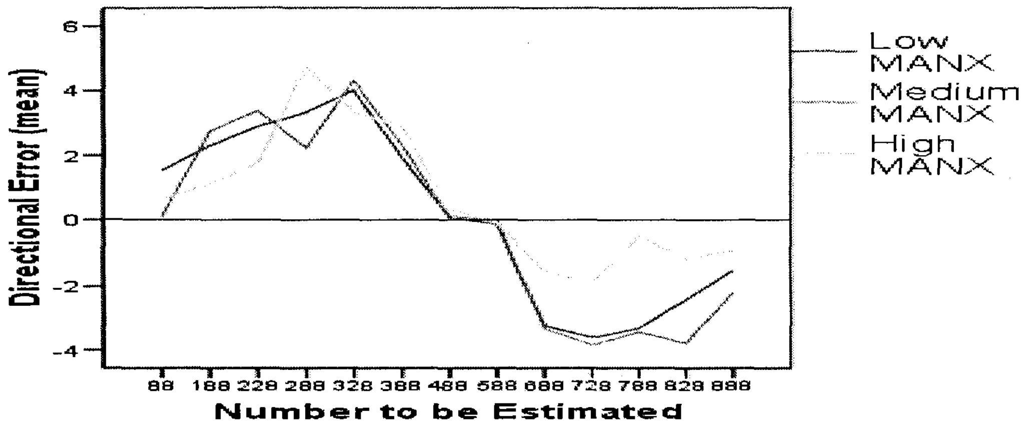

Figure 29. Directional Error by Math Anxiety Group (A)

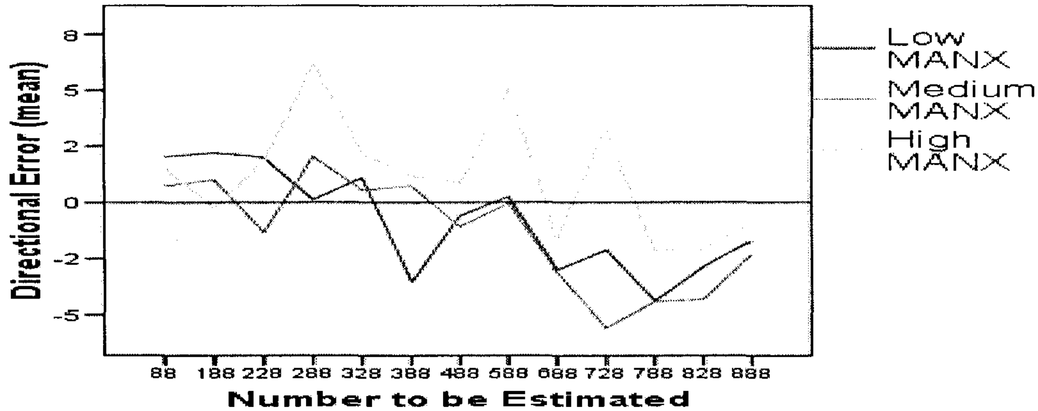

Figure 29. Directional Error by Math Anxiety Group (B)

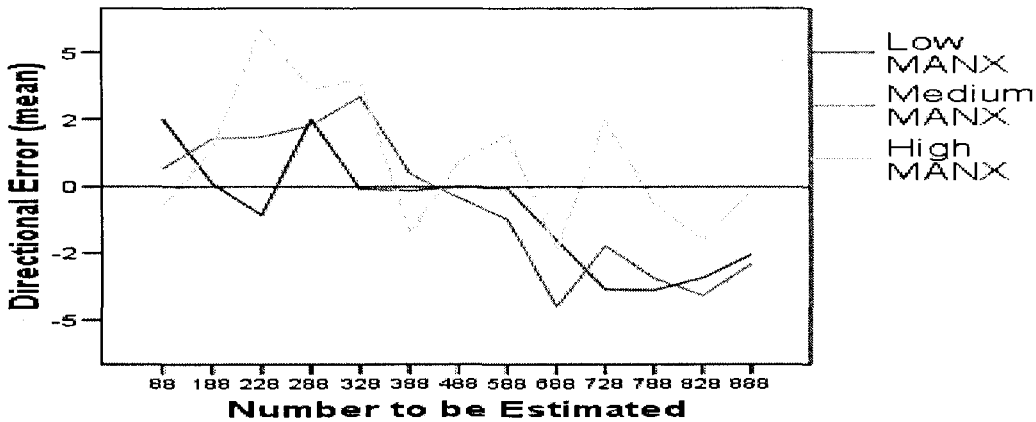

Figure 29. Directional Error by Math Anxiety Group (C)

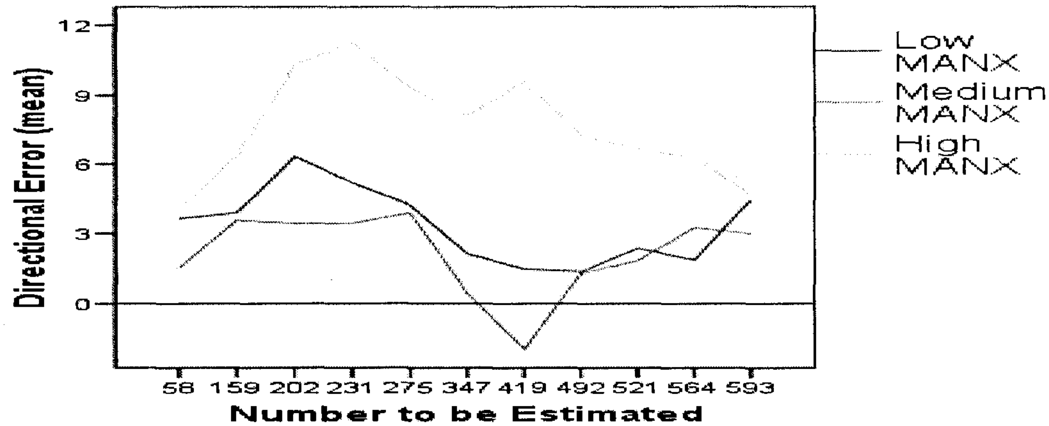

Figure 30. Directional Error by Math Anxiety Group (A)

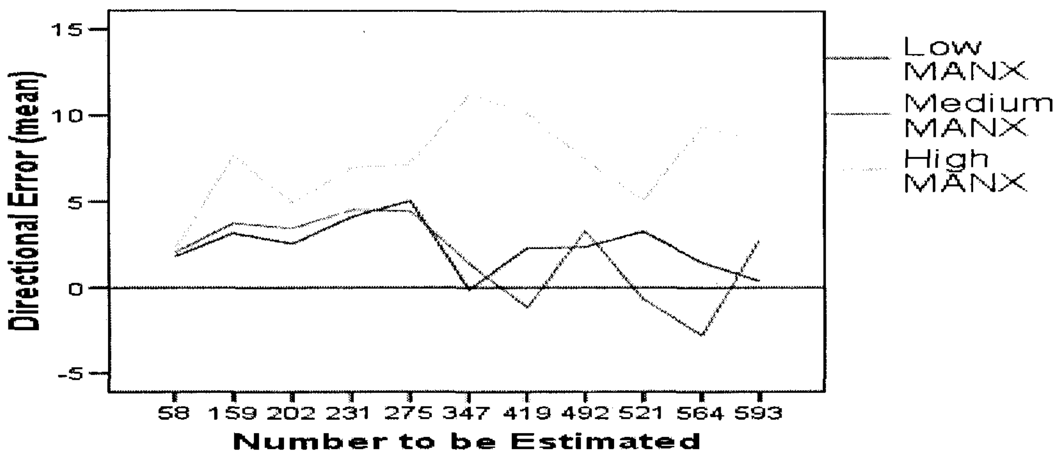

Figure 30. Directional Error by Math Anxiety Group (B)

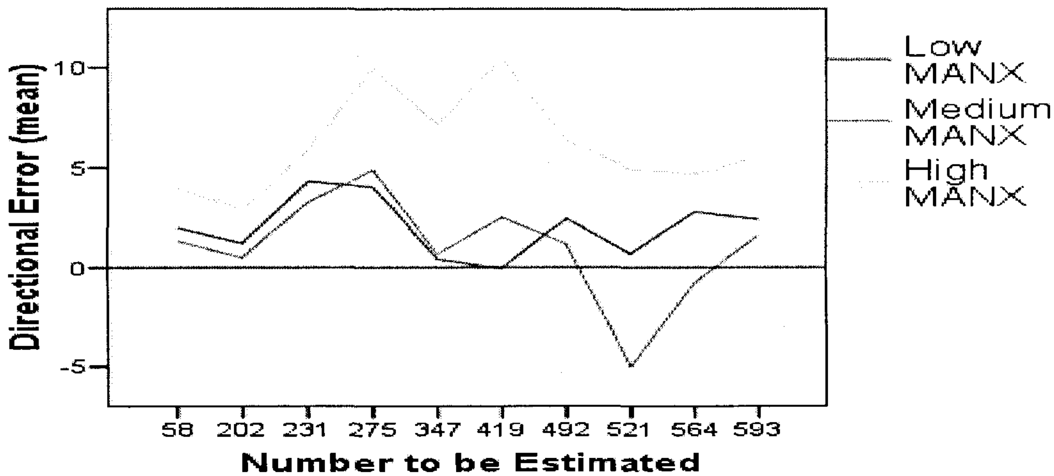

Figure 30. Directional Error by Math Anxiety Group (C)

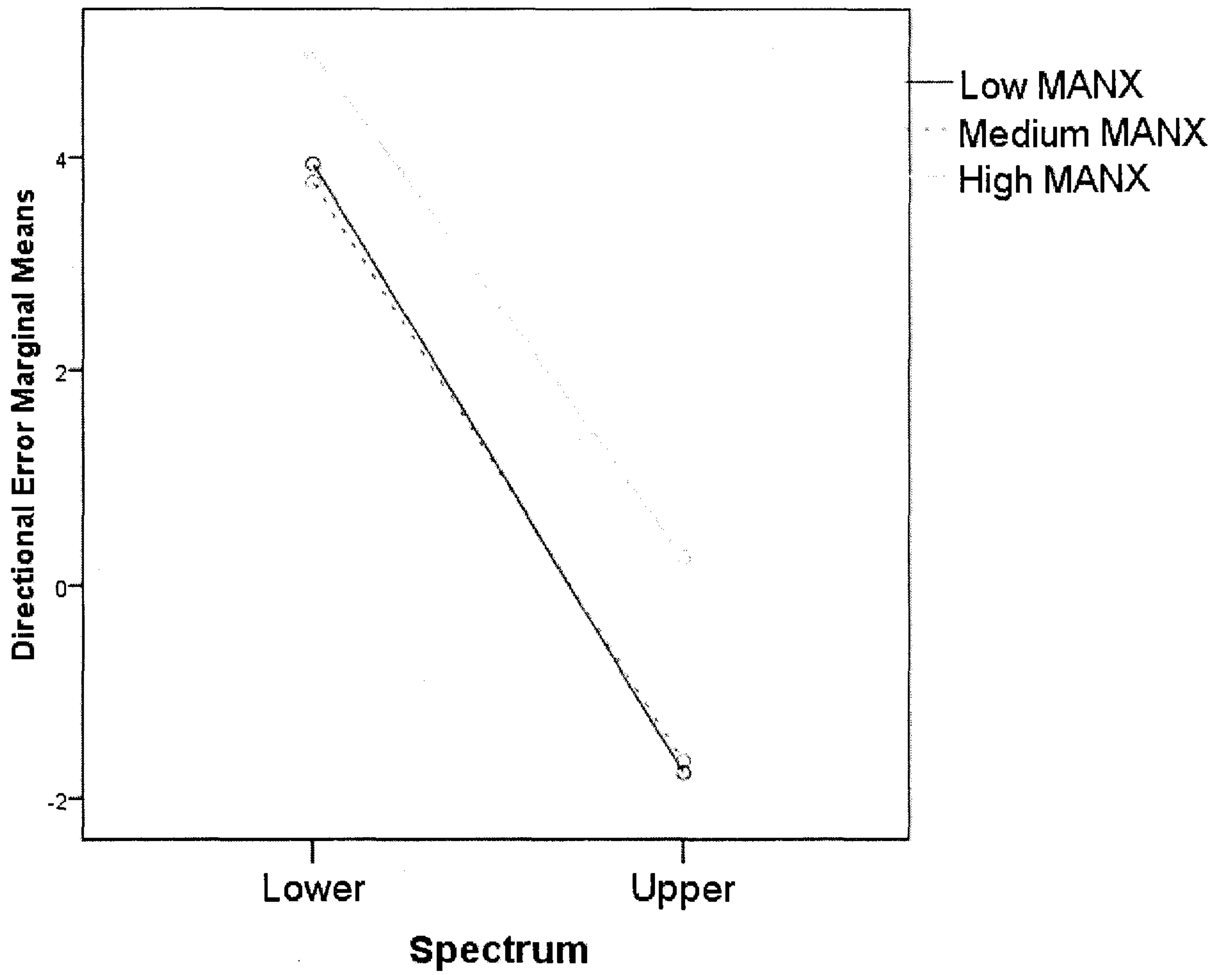

Figure 31. Directional Error by Math Anxiety Group by Spectrum, Block A (100)

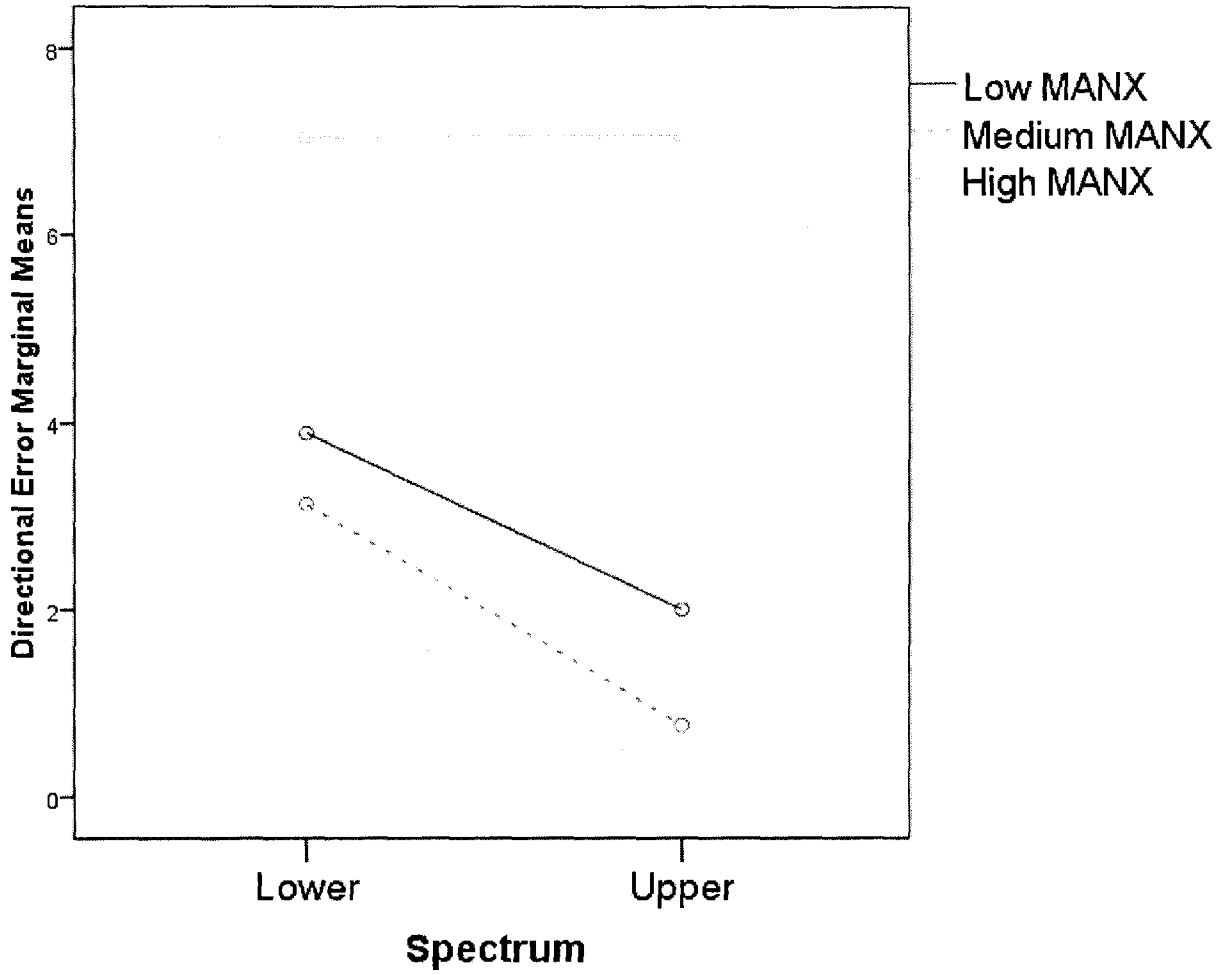

Figure 32. Directional Error by Math Anxiety Group by Spectrum, Block C (723)

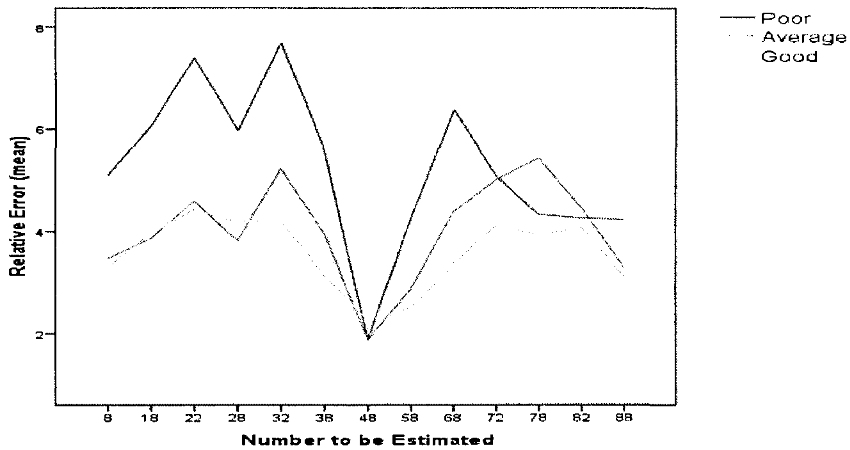

Figure 33. Relative Errors by Estimator Groups (A)

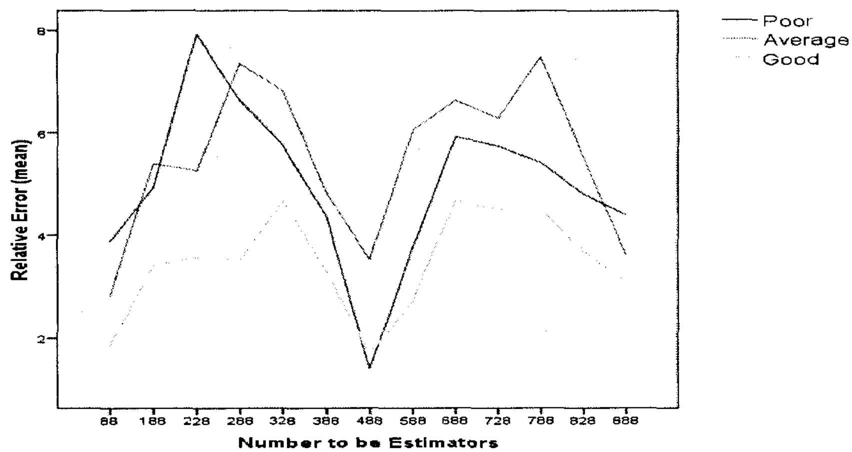

Figure 33. Relative Errors by Estimator Groups (B)

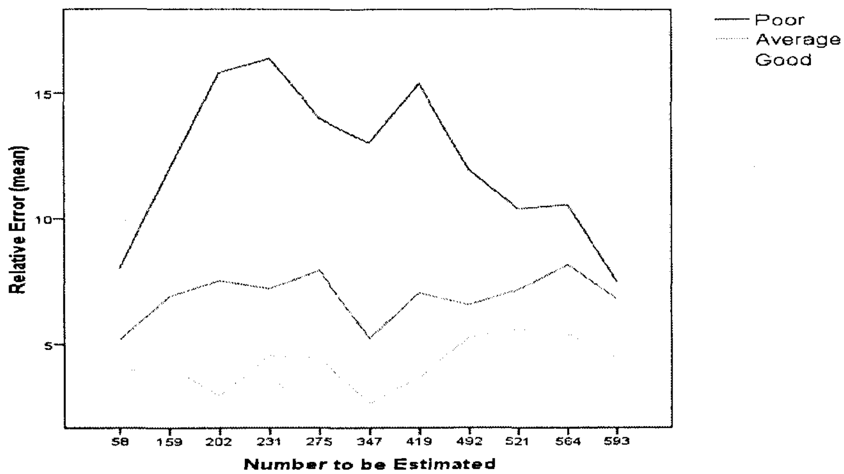

Figure 33. Relative Errors by Estimator Groups (C)

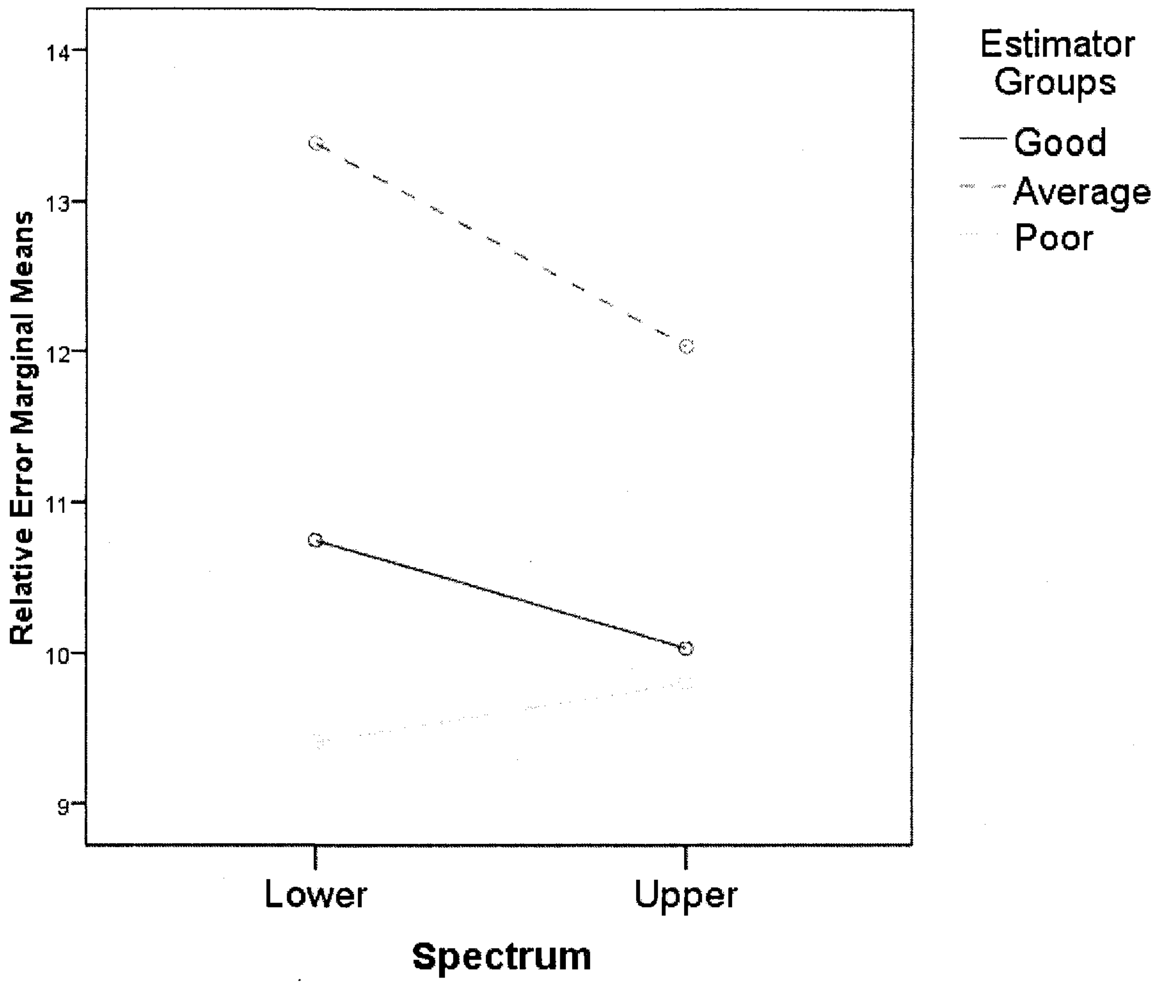

Figure 34. Relative Error by Estimator Group by Spectrum, Block A (100)

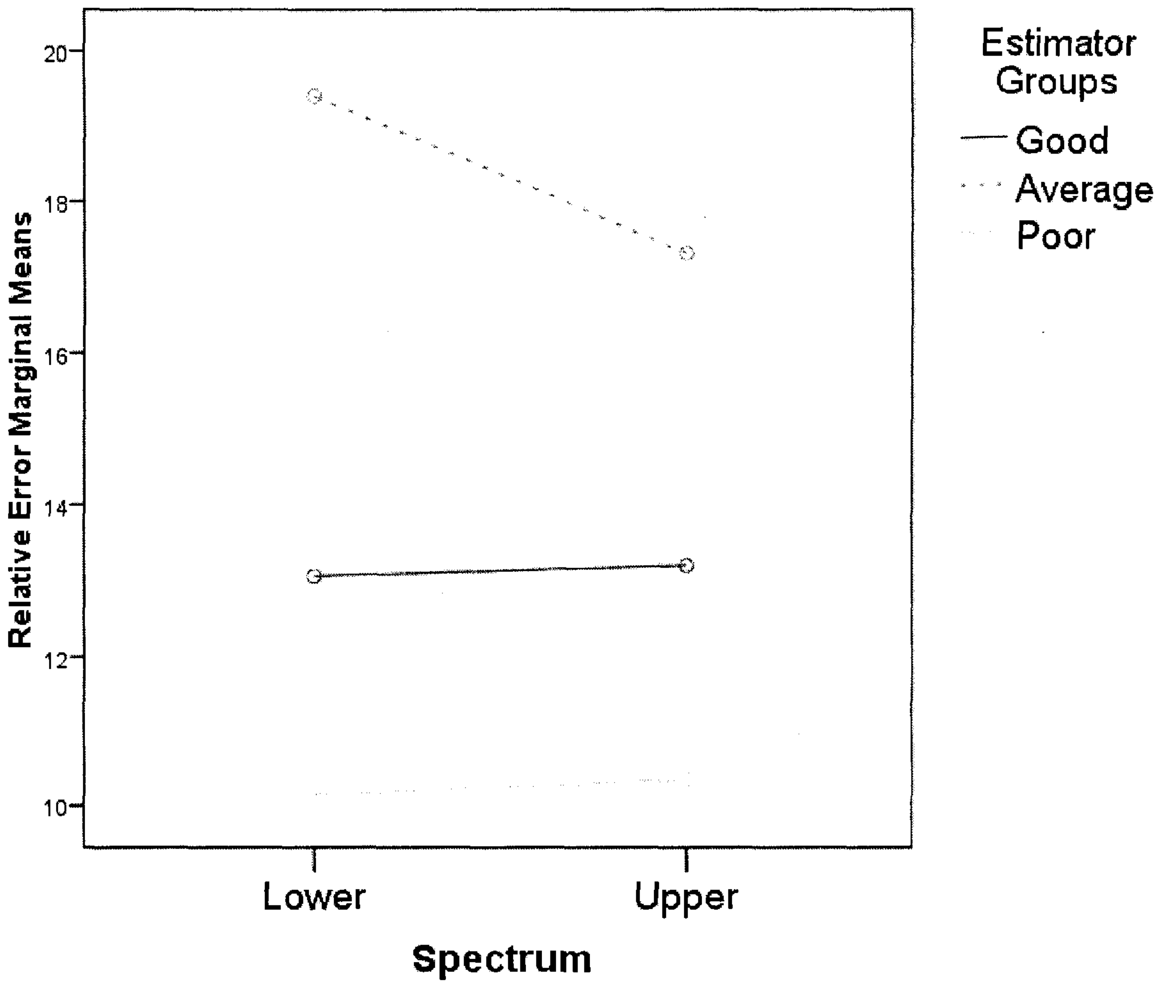

Figure 35. Relative Error by Estimator Group by Spectrum, Block C (723)

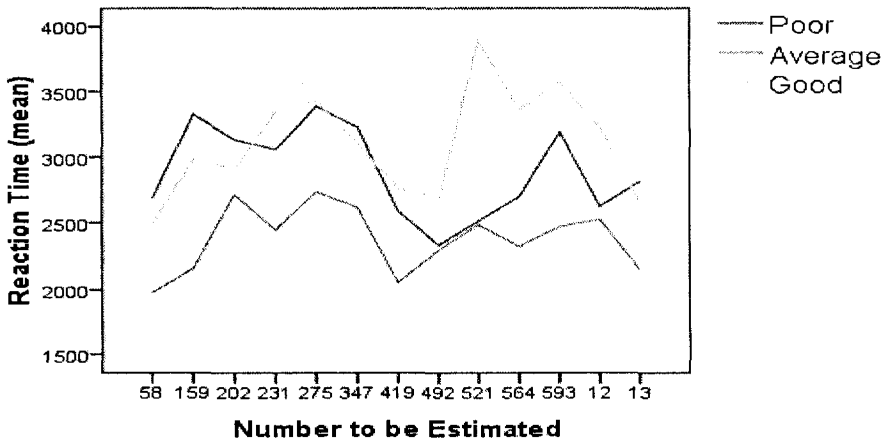

Figure 36. Reaction Times by Estimator Groups (A)

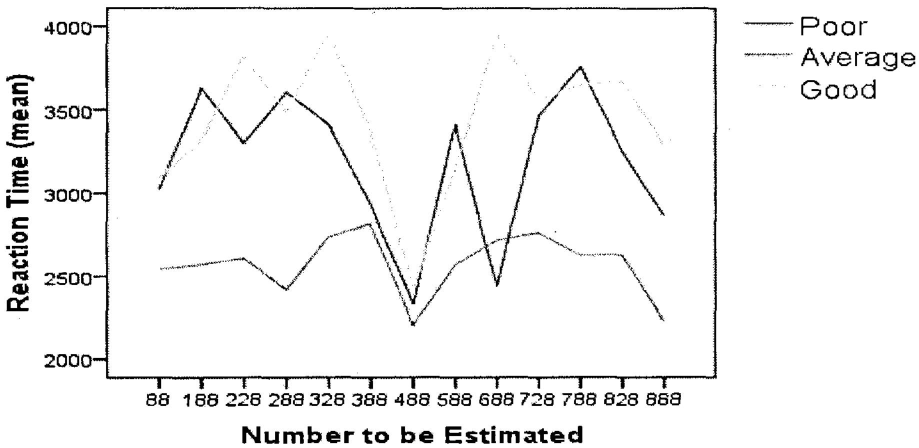

Figure 36. Reaction Times by Estimator Groups (B)

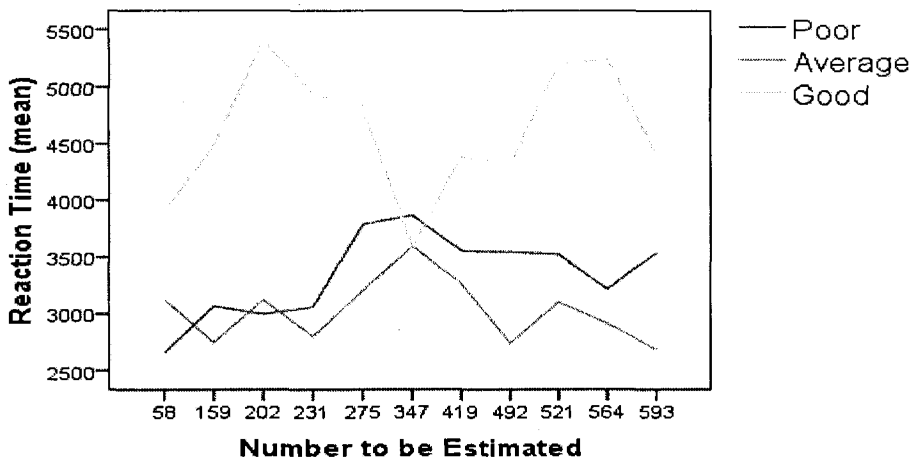

Figure 36. Reaction Times by Estimator Groups (C)

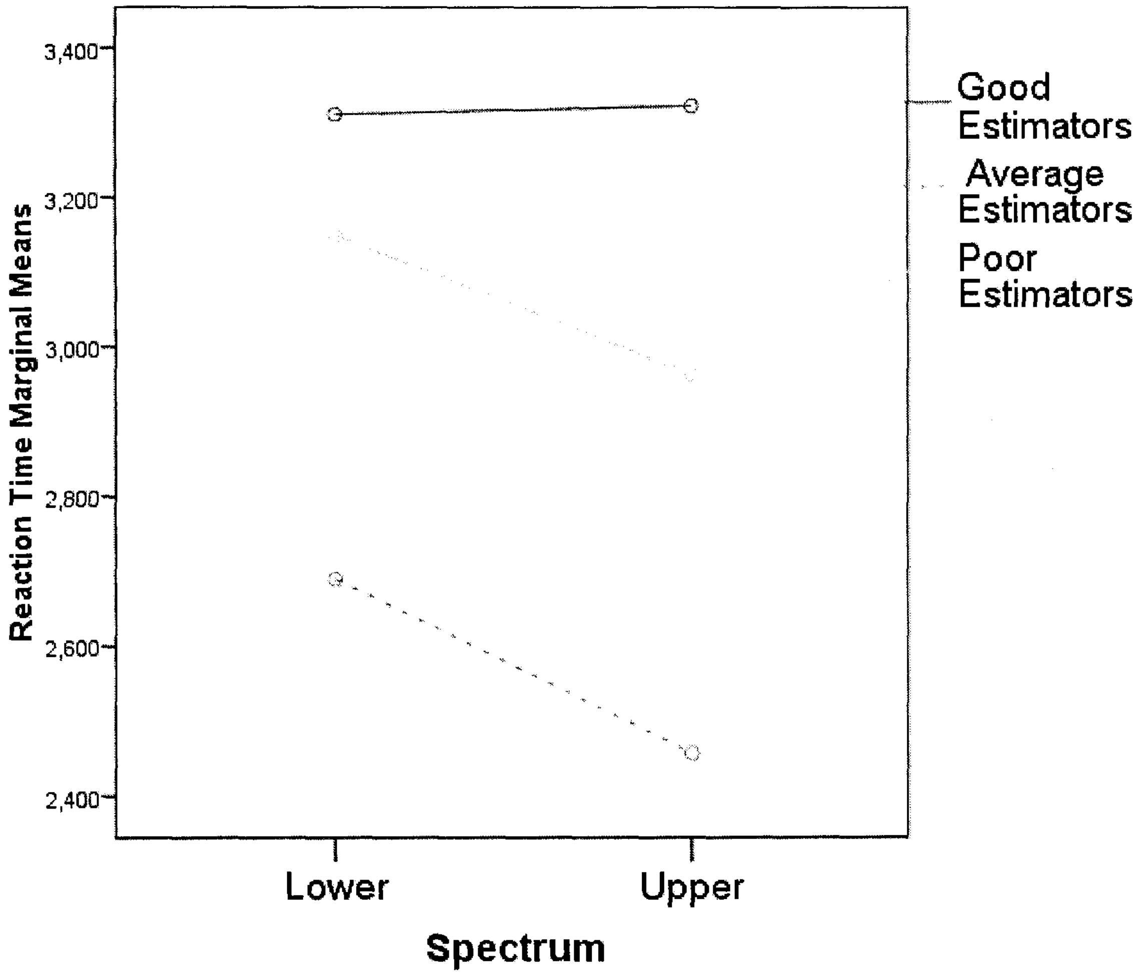

Figure 37. Reaction Time by Estimator Group by Spectrum, Block A (100)

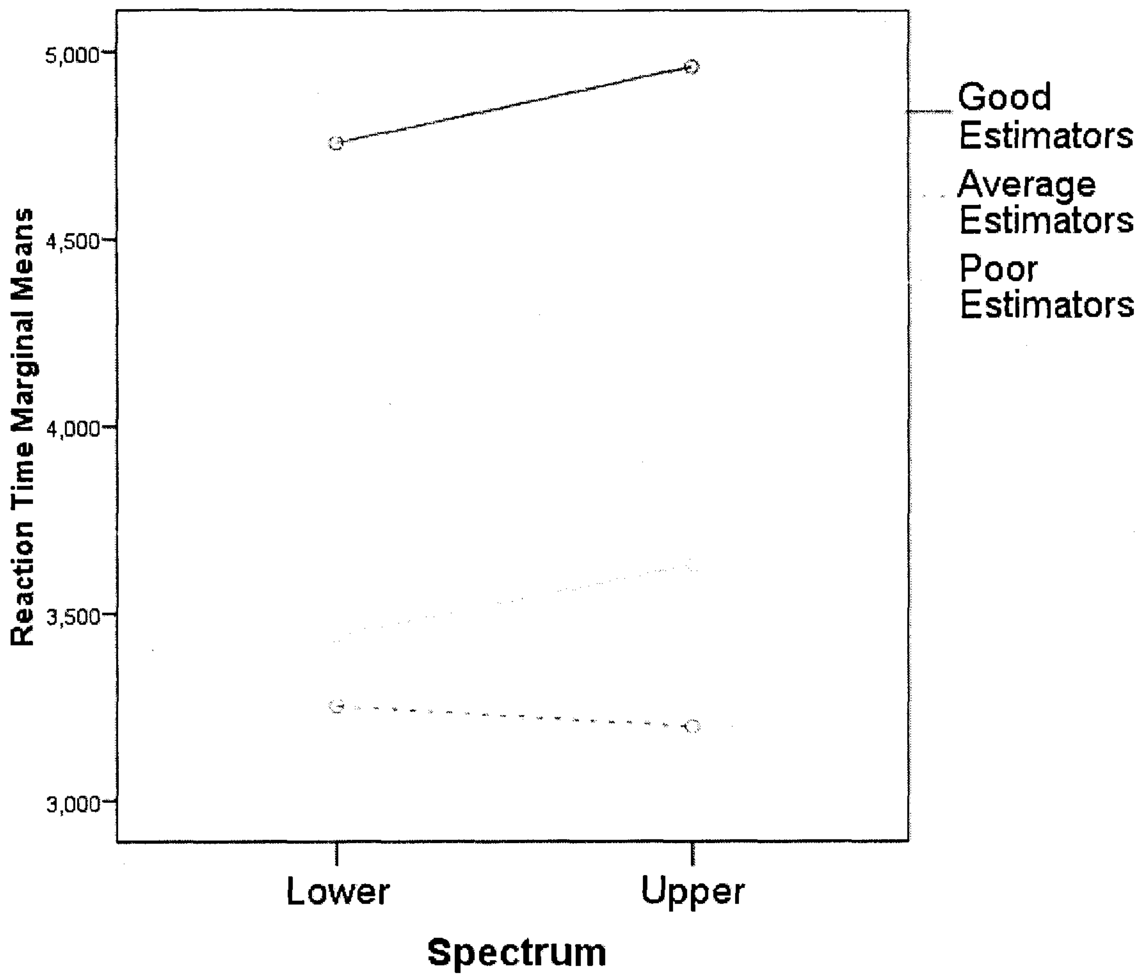

Figure 38. Reaction Time by Estimator Group by Spectrum, Block C (723)

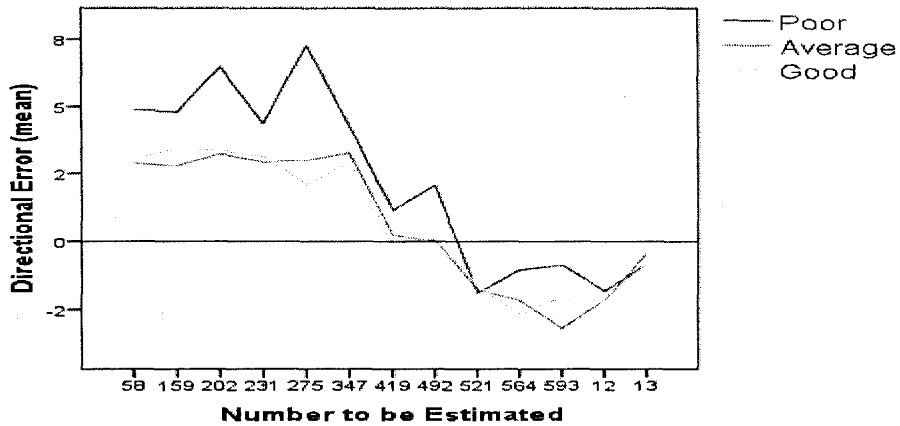

Figure 39. Directional Errors by Estimator Groups (A)

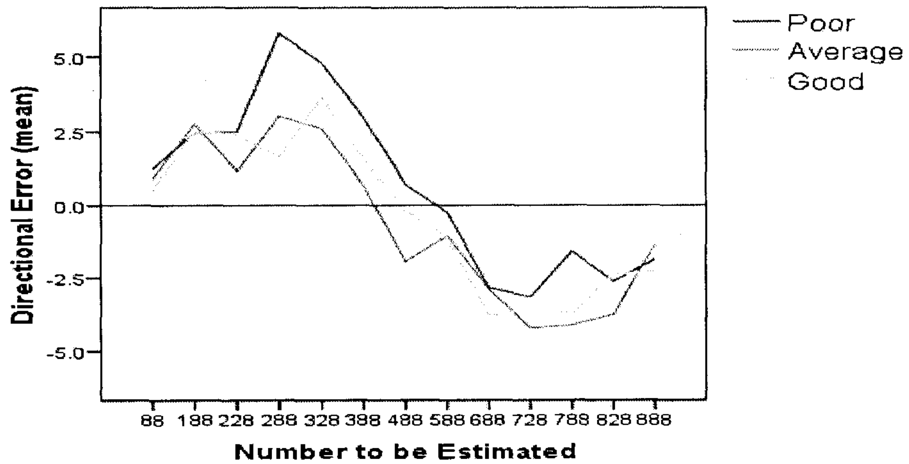

Figure 39. Directional Errors by Estimator Groups (B)

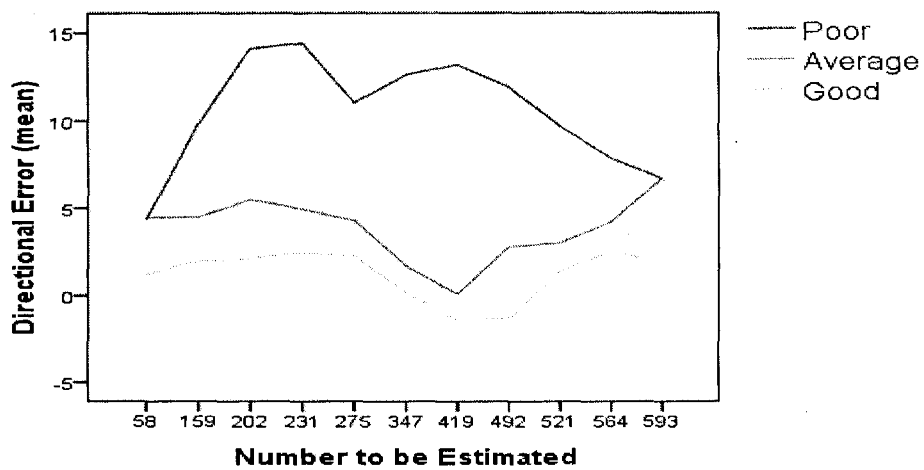

Figure 39. Directional Errors by Estimator Groups (C)

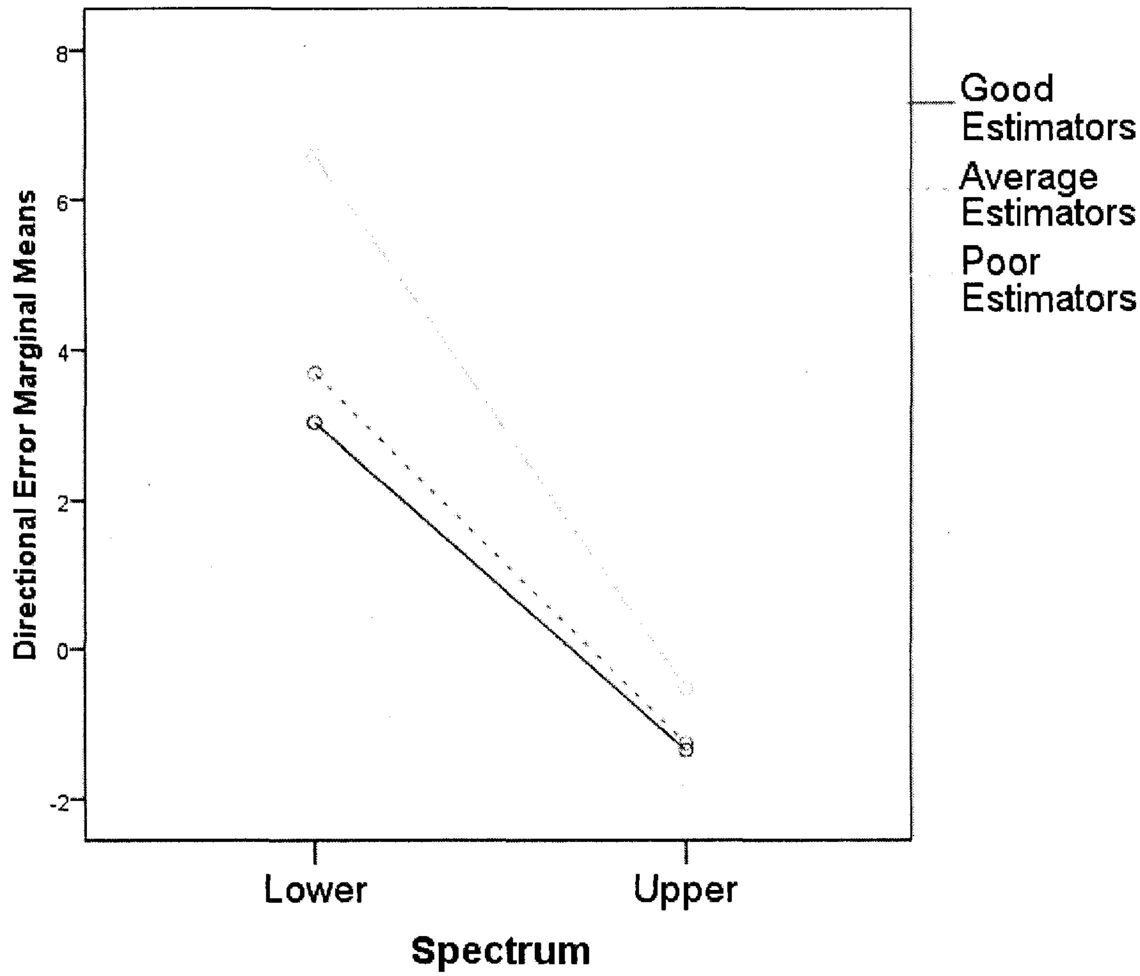

Figure 40. Directional Error by Estimator Group by Spectrum, Block A (100)

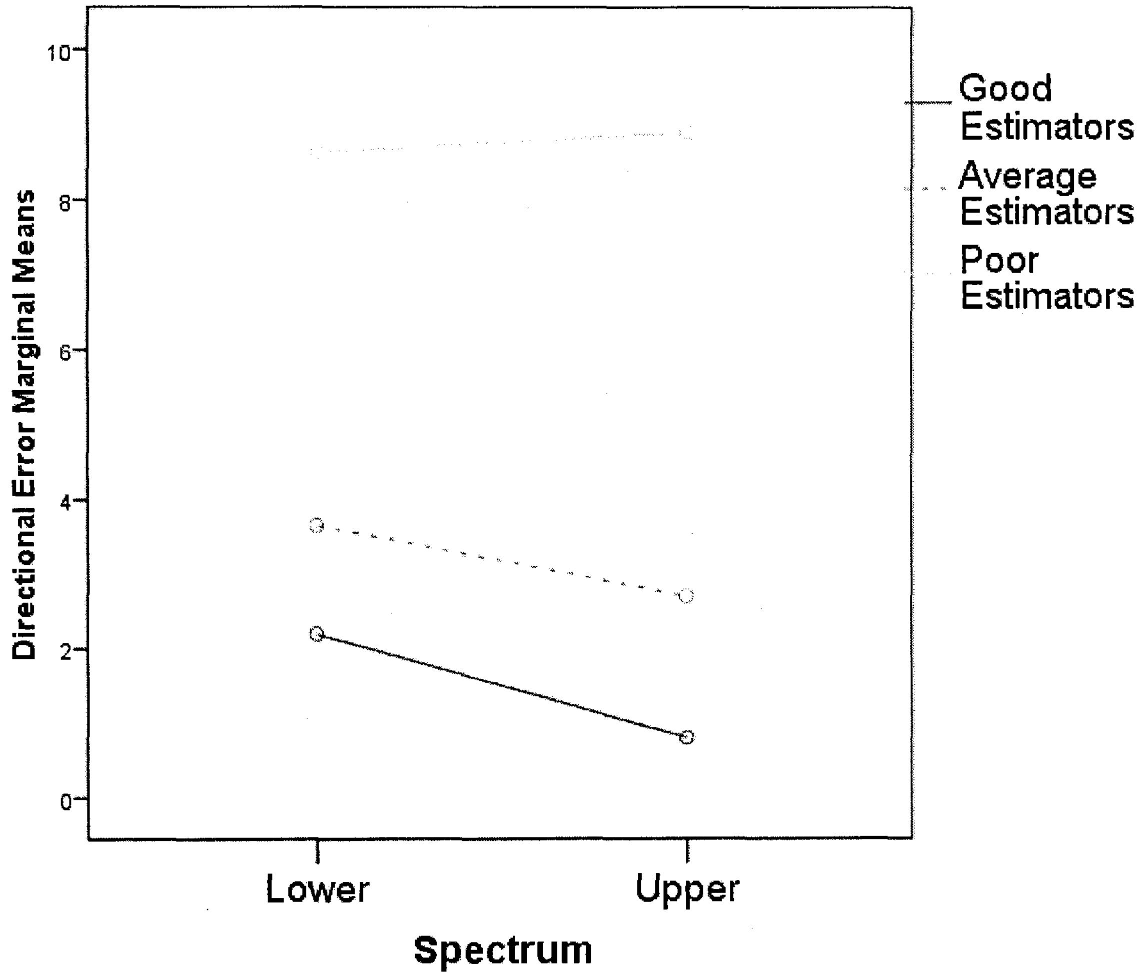

Figure 41. Directional Error by Estimator Group by Spectrum, Block C (723)

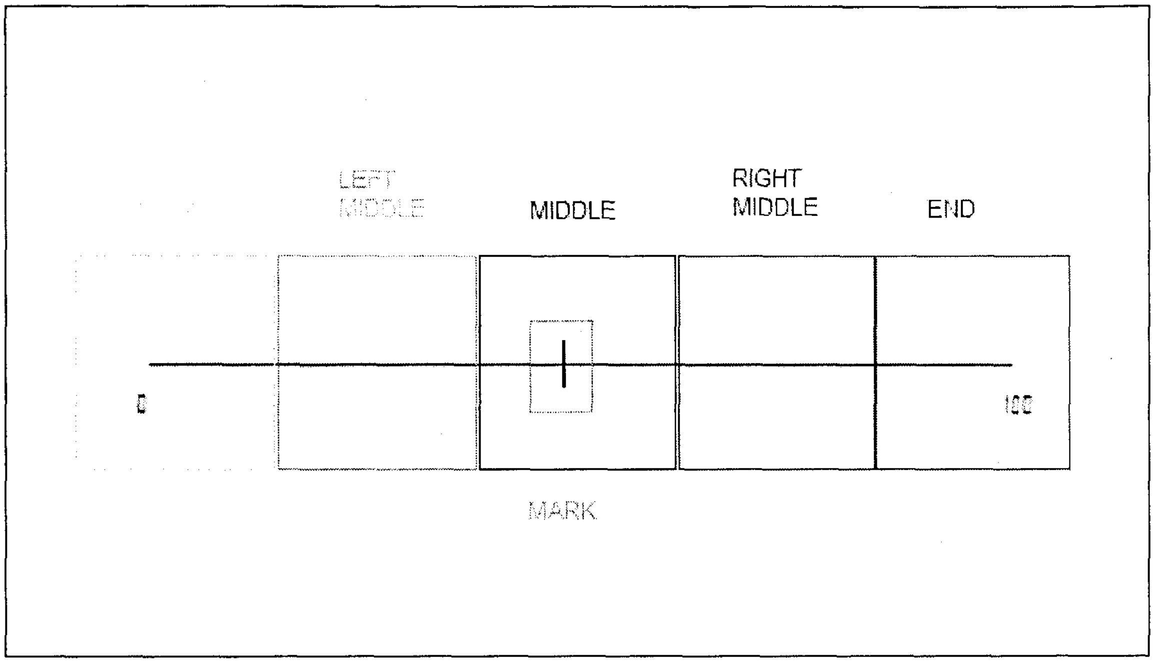

Figure 42. Example of Areas of Interest overlaid on 100 Estimation Line

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